Online Appendix
for "Coarse Revealed Preference"

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A Empirical Illustration of the Perturbation Index

We study the data collected from the portfolio choice experiment in Choi et al. (2007). The experiment was performed on 93 undergraduate subjects at UC Berkeley. Every subject was asked to make choices in 50 decision problems. In each problem, the subject divided her budget between two Arrow-Debreu securities, with each security paying 1 token (equivalent to US\$0.50) if the corresponding state was realized, and 0 otherwise. We focus on the symmetric treatment where each state of the world occurred with a commonly known probability of 1/2. This treatment was applied to 47 subjects (subject ID 201-219 and 301-328). The prices of the Arrow-Debreu securities were chosen at random (over some compact interval) and varied across problems and subjects, with income normalized at 1 throughout.

For each state $s \in \{1, 2\}$, let x_s denote the demand for the security that pays off in that state and let p_s denote its price. For each subject and in each decision problem $t \in T = \{1, \ldots, 50\}$, the state prices $p^t = (p^t_1, p^t_2)$ were randomly chosen and the subject faced a budget set

$$L(p^t) = \{x \in \mathbb{R}^2_+ : p^t_1 x_1 + p^t_2 x_2 \leq 1\}.$$

Thus the set of observations collected from a subject can be written as $\mathcal{D} = \{(x^t, L(p^t))\}_{t=1}^{50}$, where x^t is the subject's choice in $L(p^t)$.

In calculating the perturbation index, we apply Algorithm I on $\mathcal{O}^\kappa$ for different values of κ; the index can then be obtained by binary search. The algorithm for determining if $\mathcal{O}^\kappa$ is coarsely rationalizable involves calculating $T^1 := \Phi(T)$, the set of revealed dominated observations in T and then checking whether $T^1 = T$; if not, it calculates $T^2 := \Phi(T^1)$, the set of revealed dominated observations in T_1, and checks whether $T^2 = T^1$; and so forth. The sequence of nested sets T^1, T^2, $T^3 \ldots$ either ends with $T^k = T^{k-1}$ (in which case $\mathcal{O}^\kappa$ fails to satisfy NCP) or $T^k = \emptyset$ (in which case $\mathcal{O}^\kappa$ satisfies NCP). The calculation of Φ involves calculating Φ^1, Φ^2 and so forth; we ascertain whether or not a given observation is in $\Phi^m(T)$ by solving the appropriate system of linear inequalities (see Sections 2.4 and 4.1).

As an illustration, Table 1 shows the steps involved when Algorithm I is applied to Subject 201's data. The number of elements in set of the nested sequence T^1, T^2, $T^3 \ldots$ is indicated in Table 1. We see that when $\kappa = 0.2$, $\mathcal{O}^\kappa$ fails NCP because $T^{11} = T^{10}$ and is nonempty while $\mathcal{O}^\kappa$ satisfies NCP when $\kappa = 0.3$, because T^{16} is empty. Thus κ^* lies between 0.2 and 0.3. Indeed, by binary search, we find that $\kappa^* = 0.2151$.

	$\kappa = 0.2$	$\kappa = 0.3$
$\|T^1\|$	48	47
$\|T^2\|$	47	46
$\|T^3\|$	45	44
$\|T^4\|$	41	39
$\|T^5\|$	39	36
$\|T^6\|$	37	31
$\|T^7\|$	33	25
$\|T^8\|$	27	20
$\|T^9\|$	21	18
$\|T^{10}\|$	18	16
$\|T^{11}\|$	18	13
$\|T^{12}\|$		8
$\|T^{13}\|$		4
$\|T^{14}\|$		2
$\|T^{15}\|$		1
$\|T^{16}\|$		0

Table 1: Testing NCP on Subject 201.

Similar calculations are carried out for the other 46 subjects. We find that 12 subjects pass the test exactly and so have an index of 0. The median value of the index is 0.0778, with 0.1504 and 0 being the 75th and 25th percentiles respectively. The cumulative distribution of the perturbation index (κ^*) is depicted in Figure 5. For each $r \in [0, 1]$, we plot the percentage of subjects whose index are less than or equal to r.

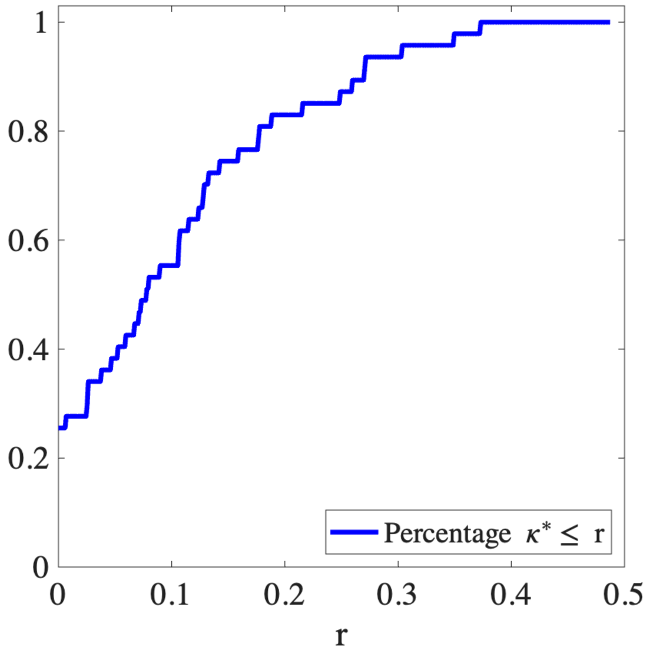

Figure 5: Distribution of κ^*.

How does the index compare with Afriat's CCEI? Since the CCEI is increasing with rationality while the opposite holds for the perturbation index, the two indices are naturally negatively correlated. We find that the rank correlation coefficient is -0.919 while the linear correlation coefficient is -0.795 (using the CCEI calculated for these subjects in Polisson, Quah and Renou (2020)). So the two indices are correlated, but not perfectly. In empirical work where conclusions are drawn based on the CCEI, one could use the perturbation index as a way of checking the robustness of those conclusions. For example, there is evidence that rationality, as measured by the CCEI, can be helpful in explaining broader economic outcomes, including family wealth (see Choi et al. (2014)); it is interesting to check whether conclusions such as these are sensitive to the rationality index used.

B Perturbation Index and Varian (1985)

The idea of relaxing a revealed preference test to allow for measurement error was also explored by Varian (1985). That paper uses this idea to test the hypothesis that a firm is cost minimizing. Using our language, the introduction of measurement error leads to a coarsening of the dataset, where the true choice made by the firm is allowed to be in a ball around the observed choice x^t. Because the hypothesis being tested is different (cost-minimization rather than utility-maximization), the test used in that paper is *not* related to the never-covered property. To be precise, Varian (1985) assumes that the observer has information on factor prices, factor demand (imperfectly observed) and the output level. In the context of consumption data, the analog to the output level would be the utility level, but notice that we do not require this information (even in an ordinal form) to be part of our observations. It is the absence of this information that makes testing the utility-maximization hypothesis (with or without coarsening) a different exercise from testing cost-minimization.

C Variations on the Perturbation Index

The perturbation and price misperception indices share a characteristic with Afriat's CCEI: in all three cases the permissible 'deviation' to restore rationality is *uniform* across all observations. In the case of the perturbation index, the bound of κ is uniformly applied to all observed bundles (see (10)); in the price misperception index, the price misperception is measured by δ and is uniformly bounded between $1/(1-\epsilon)$ and $1-\epsilon$ at each observation (see (11)); lastly, in the CCEI, a common cost efficiency e is imposed on all observations. Varian (1990) proposes a modification of the CCEI where the cost efficiency level at different observations are allowed to vary and a dataset's departure from rationality is measured by some average of the cost efficiency across all observations. A similar idea can be applied to both the perturbation and price misperception indices.

We explain this more carefully in the context of the perturbation index. Let

$$A^{t,\kappa^t} := \left\{ x \in L(p^t) : \; p^t \cdot x = 1 \text{ and } |p_i^t x_i - p_i^t x_i^t| \leq \kappa^t \text{ for all } i \right\},$$

where $\kappa^t \in [0,1]$. (This definition coincides with that given in (10)) except that the bound κ^t is allowed to vary with the observation t.) For any $\boldsymbol{\kappa} = (\kappa^1, \kappa^2, \ldots, \kappa^T)$, we can test, via NCP, whether there is a regular utility function $U : \mathbb{R}^n_+ \to \mathbb{R}$ that coarsely rationalizes

$$\mathcal{O}^{\boldsymbol{\kappa}} = \left\{ \left(A^{t,\kappa^t}, L(p^t) \right) \right\}_{t \in T}.$$

Let $S : [0,1]^T \to \mathbb{R}$ be an aggregation rule defined on different values of $\boldsymbol{\kappa}$. For example, if we set $S(\boldsymbol{\kappa}) = \sum_{t=1}^T \kappa^t / T$, then we are simply taking the arithmetic average over κ^t. Based on S, we have a *generalized perturbation index*

$$S^* := \inf\{S(\boldsymbol{\kappa}) : \mathcal{O}^{\boldsymbol{\kappa}} \text{ is coarsely rationalizable by a regular utility function}\}$$

This index corresponds to the (standard) perturbation index if $S(\boldsymbol{\kappa}) = \max_{t \leq T} \kappa^t$.

Calculating the index S^* is computationally more intensive than calculating the (standard) perturbation index κ^*. Checking if $\mathcal{O}^{\boldsymbol{\kappa}} = \{A^{t,\kappa^t}, L(p^t)\}_{t \in T}$ satisfies NCP is straightforward using our algorithm; the fact that the bounds κ^t are allowed to vary with t does not create any difficulty. However, finding S^* requires searching over $\{\kappa^t\}_{t \in T}$, which lives in a T-dimensional space.[1] This computational difficulty is exactly analogous to the difficulty involved in calculating Varian's index. Different approaches have been developed that are effective in calculating Varian's index (see Halevy, Persitz and Zrill (2018)) (by searching through the T-dimensional space more efficiently) and those approaches could potentially be used to calculate S^*.

D Calculating the Price Misperception Index (PMI)

Recall from Section 4.2 of the paper, that calculating PMI($\mathcal{D}$) requires checking the dual coarse rationalizability of $\mathcal{O}^*_\epsilon = \left\{ (x^t, \{L(\hat{p}^t)\}_{\hat{p}^t \in Z_{\delta,\epsilon}(p^t)}) \right\}_{t \in T}$, where for each t,

$$Z_{\delta,\epsilon}(p^t) := \left\{ \hat{p}^t \in \mathbb{R}^n_{++} : \delta(p^t, \hat{p}^t) \leq \frac{1}{1-\epsilon} \text{ and } x^t \cdot \hat{p}^t = 1 \right\}.$$ [2]

By Theorem 3, this is equivalent to the coarse rationalizability of

$$\mathcal{O}^{**}_\epsilon = \left\{ (Z_{\delta,\epsilon}(p^t), L(x^t)) \right\}_{t \in T},$$

[1] Note that this is unlike the case of calculating κ^* where we can require κ^t to be equal across observations with no loss of generality, effectively reducing the search to a one-dimensional problem.

[2] $\delta(p, g) = \max_{i,j \in \{1,\ldots,n\} : i \neq j} \left\{ \frac{p_i/p_j}{g_i/g_j}, \frac{g_i/g_j}{p_i/p_j}. \right\}$.

where $L(x^t) := \{p \in \mathbb{R}^n_+ : x^t \cdot p \leq 1\}$, which is in turn equivalent to checking that $\mathcal{O}^{**}_\epsilon$ obeys NCP.

Using Algorithm I, we know that NCP can be checked in polynomial time. Indeed, the procedure outlined in Section 2.4 applies here, mutatis mutandi. In particular, suppose that we want to check whether $\Phi(T') = T'$ for a nonempty subset $T' \subseteq T$ and assume that we have obtained $\Phi^k(T')$ with $\Phi^k(T') \subsetneq T'$. Then the set $\Phi^{k+1}(T')$ is given by

$$\Phi^{k+1}(T') = \left\{ t \in T' : Z_{\delta,\epsilon}(p^t) \subseteq \left(\bigcup_{t \in T'} \mathring{L}(x^t) \right) \bigcup \left(\bigcup_{t \in \Phi^k(T')} L(x^t) \right) \right\}.$$

To determine whether some given $s \in T'$ belongs to $\Phi^{k+1}(T')$, we can check if there is a solution $p \in \mathbb{R}^n$ to the following system of inequalities which, crucially, are linear in p:

$$\begin{array}{rcll} p & \geq & 0 & \\ x^s \cdot p & = & 1 & \\ (1-\epsilon) p_j \cdot \dfrac{p^s_i}{p^s_j} & \leq & p_i \leq \dfrac{p_j}{1-\epsilon} \cdot \dfrac{p^s_i}{p^s_j} & \text{for all goods } i \neq j \\ x^t \cdot p & \geq & 1 & \text{for all } t \in T' \\ x^t \cdot p & > & 1 & \text{for all } t \in \Phi^k(T'). \end{array}$$

Clearly, $t \in \Phi^{k+1}(T')$ if and only if there is *no* solution to this system of linear inequalities.

E An Alternative Check for Coarse Rationalizability

In Section 2.4 of the main paper, we explain how the never-covered property can be used to check in polynomial time if the dataset $\mathcal{O} = \{(A^t, B^t)\}_{t \in T}$ admits a coarse rationalization, in the special case where $B^t = L(p^t) = \left\{ x \in \mathbb{R}^n_+ : p^t \cdot x \leq 1 \right\}$ and

$$A^t = \left\{ x \in \mathbb{R}^n_+ : p^t \cdot x = 1 \text{ and } C^t \cdot x \leq c^t \right\}$$

(with $C^t \in \mathbb{R}^{k \times n}$ and $c^t \in \mathbb{R}^k$). We now present an alternative approach for verifying the coarse rationalizability of $\mathcal{O}$ using the Afriat inequalities.

Afriat's Theorem (see Afriat (1967)) states that the following three statements are equivalent: (i) a dataset $\mathcal{D} = \{(x^t, L(p^t))\}_{t \in T}$ is rationalizable; (ii) $\mathcal{D}$ satisfies

GARP; (iii) there are u^t and λ^t (for $t \in T$) that solve the following *Afriat inequalities*:

$$\begin{aligned} \lambda^t &> 0 \quad \text{for all } t, \\ u^s &\leq u^t + \lambda^t p^t \cdot (x^s - x^t) \quad \text{for all } s \neq t. \end{aligned}$$

Solving the system in (iii) is straightforward as it is just a system of linear inequalities, and so it provides a computationally viable method of checking if $\mathcal{D}$ admits a rationalization, just as checking GARP is also computationally straightforward.

It follows immediately from Afriat's Theorem that the dataset $\mathcal{O} = \{(A^t, B^t)\}_{t \in T}$ admits a coarse rationalization if and only if there are u^t, λ^t, and bundles $z^t \in \mathbb{R}^n_+$ (for all $t \in T$) that solve

$$\begin{aligned} C^t \cdot z^t &\leq c^t \\ p^t \cdot z^t &= 1 \\ z^t &\geq 0 \\ \lambda^t &> 0 \\ u^s &\leq u^t + \lambda^t p^t \cdot (z^s - z^t) \quad \text{for } s \neq t. \end{aligned} \tag{1}$$

However, this approach to checking the coarse rationalizability of $\mathcal{O}$ is not satisfactory because the system of inequalities is now bilinear in the unknowns and solving a bilinear system is, in general, an NP-hard problem (see Toker and Ozbay (1995)).

We now describe an alternative way of checking coarse rationalizability using a dual version of the Afriat inequalities which leads to a linear system. This approach was suggested to us by an anonymous referee.

First, we note that it follows from Afriat's Theorem that $\mathcal{O}$ is coarsely rationalizable if one can select $z^t \in A^t$ for each $t \in T$ such that the induced data set $\{(z^t, B^t)\}_{t \in T}$, where $B^t = L(p^t)$, satisfies GARP. Since $p^t \cdot z^t = 1$ for each t, we have, for all $t, s \in T$, z^t is directly revealed (strictly) preferred to z^s if $1 = p^t \cdot z^t \geq (>)\, p^t \cdot z^s$. It is clear that, formally, the roles of p^t and z^t are interchangeable; thus, if we define p^s as directly revealed (strictly) preferred to p^t if $1 \geq (>) p^t \cdot z^s$, then there is a revealed preference cycle over bundles if and only if there is a revealed preference cycle over price vectors. It follows from this observation that $\{(z^t, B^t)\}_{t\in T}$ satisfies GARP if and only if $\widetilde{\mathcal{O}} = \{(p^t, \tilde{B}^t)\}_{t\in T}$, where $\tilde{B}^t = L(z^t)$, satisfies GARP.[3] The latter is in turn equivalent to the existence of a solution to the corresponding Afriat inequalities.

To summarize, a necessary and sufficient condition for coarse rationalizability of $\mathcal{O} = \{(A^t, B^t)\}_{t\in T}$ is the existence, for each $t \in T$, of z^t, λ^t, and V^t such that, for all $t, s \in T$,

$$\begin{aligned} C^t \cdot z^t &\leq c^t, \\ z^t \cdot p^t &= 1, \\ z^t &\geq 0, \\ \lambda^t &> 0, \\ V^s &\leq V^t + \lambda^t z^t \cdot (p^s - p^t). \end{aligned}$$

Defining $\mu^t = \lambda^t z^t$, this is equivalent to the existence, for each $t \in T$, of μ^t, λ^t, and V^t such that

$$\begin{aligned} C^t \cdot \mu^t &\leq \lambda^t c^t, \\ \mu^t \cdot p^t &= \lambda^t, \\ \mu^t &\geq 0, \\ \lambda^t &> 0, \\ V^s &\leq V^t + \mu^t \cdot (p^s - p^t). \end{aligned}$$

Crucially, unlike the system of inequalities in (1), this system is linear in the unknowns.

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[3]Recall that the same trick is used in the proof of Theorem 3 in the main paper.

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