When Autarky Trumps Free and Costless Trade[*,†]

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November, 2025

Abstract

In the context of Krugman's (1979) canonical New Trade model, we demonstrate that a country is better off in autarky than in free and costless trade, when the productivity of its trading partner is sufficiently low.

Keywords: Gains from Trade, Trade Costs, Tariffs. New Trade Theory, Monopolistic Competition, Love of Variety, Scale Gains, Choke Prices.

JEL Codes: F12, F13.

[*]We would like to thank, without implicating, Mary Amiti, Arnaud Costinot, Ernesto Dal Bo, Ron Davies, Avinash Dixit, Dave Donaldson, Ehsan Ebrahimy, Rupert Gatti, Paul Klemperer, Flavien Moreau, John Morrow, Peter Neary, Stephen Redding, Bob Staiger, Tony Venables, and Tim Willems, as well as seminar audiences at the NBER, Ifo Institute, Harvard Business School, the University of Munich, Loughborough University, the IMF, Oxford University and University College Dublin. We are also grateful to the co-editor of this journal, Pierre-Olivier Weill, and two anonymous referees for comments and suggestions that have greatly improved the paper.

[†]Opinions expressed in this paper are those of the authors and should be reported as such. They may not reflect the views of the institutions the authors are affiliated with.

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1 Introduction

Kicking off New Trade Theory, Krugman (1979) demonstrated that increasing returns and a taste for variety enable Pareto-improving international trade, even in the absence of comparative advantages. Specifically, he established that symmetric countries are better off under free and costless trade than in autarky. In this paper we explore the contingency of this seminal finding. Relaxing symmetry, we show that a country, A, is strictly better off in autarky than under free and costless trade, when the productivity of its trading partner, B, is low. Here, 'free trade' refers to the absence of revenue-generating tariffs, 'costless trade' refers to the absence of (non-revenue-generating) iceberg trade costs, and 'productivity' refers to the reciprocal of marginal costs.

Our argument hinges on two key observations. When choke prices are finite:

1. Trade increases the scale of production, reducing the number of domestic varieties.
2. Consumer surplus is enjoyed only on infra-marginal units of a variety, with surplus increasing approximately quadratically with consumption.

These elements interact as follows: When productivity in country B is low, its firms export little, curbing the consumption of infra-marginal units of foreign varieties in A. As a result, country A's gains from trade are small, becoming second-order in the limit. Meanwhile, households in A consume many infra-marginal units of domestically produced varieties, so the welfare loss from a drop in domestic varieties is first order. With first-order losses outweighing second-order gains, country A is better off in autarky than under free and costless trade when productivity in B is low.

More succinctly, in general equilibrium (GE), imports create an externality that atomistic households fail to internalize: collectively, they drive out domestic varieties, causing a first-order social loss. When B's productivity is low, this loss outweighs the private, second-order gains from trade. Therefore, country A benefits from banning trade altogether, even when trading is free and costless. While A gains from switching to autarky, B loses.

Our result hinges on the assumption of finite choke prices, making it inapplicable to CES preferences. Under finite choke prices, increasing productivity in B from zero reduces the number of domestic varieties in A while keeping the output of surviving varieties unchanged. In contrast, with CES preferences, higher productivity in B lowers the output of domestic varieties in A but leaves their number constant.[1] This distinction is critical. In the first case, A loses domestic varieties that, as a whole, generate strictly positive surplus per dollar (SPD), while gaining marginal ('first') units of new foreign varieties with only second-order SPD, because their utility equals their price.

[1] Strictly speaking, this only holds for *symmetric* CES preferences. See Section 4 for details.

Thus, A is worse off. In the second case, A sheds 'last' units of domestic varieties that yield second-order SPD but gains 'first' units of new foreign varieties that, due to the infinite choke price, yield strictly positive SPD. As a result, A is better off.

Our assumption of finite choke prices implies decreasing elasticity of substitution near zero. This is the standard case, giving rise to intuitive pro-competitive effects of market entry, such as lower prices and reduced mark-ups (see, e.g., Zhelobodko *et al.,* 2012). Under these conditions, monopolistic competition results in insufficient scale and excess variety, as firms limit supply to sustain higher prices (see, e.g., Dixit and Stiglitz, 1977). This makes our explanation for autarky trumping free and costless trade paradoxical: starting from excess variety, how can reducing variety diminish welfare in country A? The resolution lies in distinguishing between two distinct trade-offs. In autarky, optimal variety reflects a balance between variety and scale. In contrast, the preference for autarky over free and costless trade hinges on the trade-off between domestic and foreign varieties. These trade-offs are, essentially, orthogonal.

One might expect that country A would also fare better in autarky than under free and costless trade if B's fixed-cost efficiency—the reciprocal of the fixed cost of production—is sufficiently low. After all, in the limit as B's efficiency approaches zero, country A once again lives in autarky and in free and costless trade simultaneously, and a small increase in efficiency from zero initiates trade. Despite the apparent similarity with the low-productivity scenario, in this case, country A does benefit from a trade-initiating increase in efficiency. The key difference is that low productivity in B discourages scale, whereas low efficiency promotes it.

The paper is organized as follows: The remainder of the Introduction reviews the relevant literature. Section 2 presents a generalized version of Krugman (1979) that allows for asymmetries between countries. Section 3, the core of the paper, starts with a simple example before presenting the general result and its intuition. Section 4 examines CES, while Section 5 explores the effects of low fixed-cost efficiency in B. Section 6 concludes. Proofs have been relegated to the Appendix.

Related Literature Beyond Krugman (1979), the most relevant antecedents of our work are Kokovin *et al.* (2022) and Morgan *et al.* (2020). These papers, which focus on trade costs, argue that Krugman's emphasis on the dichotomy between autarky and free and costless trade is restrictive, overlooking important non-monotonicities in between. They find that when trade begins due to a fall in trade costs, welfare declines in both countries. Still, free and costless trade remains preferable to autarky. Our analysis demonstrates that focusing on symmetry is similarly restrictive: it overlooks the fact that autarky can dominate free and costless trade, at least for one country.

One may wonder why our finding, as well as the one by Kokovin *et al.* (2022) and Morgan *et al.* (2020), has gone unnoticed for so long. The key lies in a pivotal shift between Krugman (1979) and Krugman (1980): the introduction of (symmetric) CES utility. This assumption enabled closed-form solutions and became integral to most New Trade and 'New' New Trade models. However, it also introduced two peculiar features—constant markups and infinite choke prices. Infinite choke prices imply that trade occurs for all iceberg trade costs, eliminating the initiation of trade and preventing autarky from outperforming free and costless trade. Hence, the widespread reliance on CES likely concealed the existence of both types of 'Bad Trade.'

While CES utility has long dominated the trade literature, its limitations are increasingly recognized, as highlighted by Dhingra and Morrow (2019). Extending Dixit and Stiglitz (1977) to heterogeneous firms, Dhingra and Morrow compare CES with utility functions that allow for variable elasticity of substitution. Focusing on autarky, they show that the market generates optimal variety if and only if utility is CES. This is because CES leads to constant markups, making prices proportional to both marginal costs and marginal utility, which aligns with social optimality. Outside of CES, Dhingra and Morrow find that the market generates various distortions.[2] However, as we have argued, the distortion between scale and domestic variety is essentially orthogonal to the distortion between domestic and foreign varieties.

Finally, our paper builds on Venables (1982), who studies a small open economy that exports a homogeneous good under constant returns to scale, while importing differentiated goods that compete with a local monopolistically competitive industry. Assuming domestic firms cannot export, trade displaces local varieties, and Venables analyzes the welfare implications of this process. Having foreclosed general equilibrium effects, he finds that trade raises welfare if and only if the elasticity of utility of the foreign variety is lower than that of the displaced domestic variety. We uncover the same condition in general equilibrium, because marginal changes in B's productivity near zero leave the domestic price level unchanged.

2 Model

Our model generalizes Krugman (1979), allowing for general asymmetries between countries.

[2] Dhingra and Morrow distinguish between 'aligned' and 'misaligned' incentives, depending on whether the elasticities of utility and marginal utility have derivatives with the same sign. Our Lemma 1 implies that incentives are aligned when elasticities are monotone.

2.1 Setup

There are two countries, A and B. For concreteness, we take the perspective of country A. The situation for country B is the mirror image. Country A has a fixed mass $L_A > 0$ of households and a variable mass $n_A > 0$ of active firms. The mass of potentially active firms is unbounded, and market entry occurs until the marginal firm just breaks even. Rivalry between firms is monopolistically competitive. Each household inelastically supplies one unit of labor and, using labor as the only scarce input, each active firm $i_A \in [0, n_A]$ produces a differentiated good, also denoted by i_A. We refer to these differentiated goods as varieties. While labor is domestically supplied, firms can sell their goods both domestically and abroad. When exporting from A to B, a firm incurs no cost and pays no tariffs.

The utility maximization problem of a household in country A is given by

$$\begin{array}{ll} \max_{z_{i_A}, z_{i_B}} & U_A \equiv \int_{i_A=0}^{n_A} v_A [z_{i_A}] \, di_A + \int_{i_B=0}^{n_B} v_A [z_{i_B}] \, di_B \\ \text{st.} & \int_{i_A=0}^{n_A} p_{i_A} z_{i_A} di_A + \int_{i_B=0}^{n_B} s_{i_B} z_{i_B} di_B = I_A \ . \end{array} \tag{1}$$

Here, $z_{i_A}, z_{i_B} \geq 0$ denote quantities of domestic varieties i_A and foreign varieties i_B, respectively, while p_{i_A} and s_{i_B} denote their prices. Household income is I_A. The sub-utility function, $v_A [\cdot]$, is twice differentiable with $v_A [0] = 0$. Furthermore, $0 < v'_A [\cdot] < \infty$, $-\infty < v''_A [\cdot] < 0$, and $\lim_{z \to \infty} v' [z] = 0$. Finite marginal utility at zero implies that each variety i_k, $k \in \{A, B\}$, has a choke price, i.e., a finite price above which households stop consuming that variety.[3] Households have a taste for variety. To see this, notice that, jointly, $v_A [0] = 0$ and concavity of $v_A [\cdot]$ imply that $n v_A [z/n]$ is strictly increasing in n for all $n, z > 0$.

Let $\varepsilon_{v'_A}$ denote *minus* the elasticity of marginal utility v'_A with respect to z, i.e., $\varepsilon_{v'_A} [z] \equiv -z v''_A [z] / v'_A [z]$. Slightly relaxing Krugman (1979), who assumes that $\varepsilon_{v'_A}$ is strictly increasing everywhere, we assume that $\varepsilon_{v'_A}$ is non-decreasing for $z > 0$. Since $\varepsilon_{v'_A} [0] = 0$, whereas $\varepsilon_{v'_A} [\cdot]$ is strictly positive for $z > 0$, non-decreasingness implies that $\varepsilon_{v'_A} [\cdot]$ is locally strictly increasing at $z = 0$.

Let ε_{v_A} denote the elasticity of utility v_A with respect to z, i.e., $\varepsilon_{v_A} [z] \equiv z v'_A [z] / v_A [z]$. For future reference, we state the following technical lemma, whose proof can be found in the Appendix.

Lemma 1 $\varepsilon_{v_A} [\cdot]$ *is non-negative, strictly decreasing, and* $\lim_{z \to 0} \varepsilon_{v_A} [z] = 1$.

The utility maximization problem in (1) yields the following first-order conditions (FOC). For

[3] Notice, however, that a variety's choke price is not a constant; it is increasing in the prices of other varieties.

$(i_A, i_B) \in [0, n_A] \times [0, n_B]$,

$$v'_A[z_{i_A}] \overset{(\leq)}{=} \lambda_A p_{i_A} \text{ if } z_{i_A} \overset{(=)}{>} 0, \text{ and } v'_B[z_{i_B}] \overset{(\leq)}{=} \lambda_A s_{i_B} \text{ if } z_{i_B} \overset{(=)}{>} 0 \ . \tag{2}$$

Here, $\lambda_A \in (0, \infty)$ denotes the Lagrange multiplier on the budget constraint, i.e., the shadow price of income, I_A. Since the Lagrangian is equal to the marginal utility of income, the marginal price index (that is, the cost of an additional 'util') is $P_A \equiv 1/\lambda_A$, $0 < P_A < \infty$. Notice that demand for each good is only a function of its own price p_{i_A} (or s_{i_B} for foreign goods) and the marginal price index P_A. In other words, P_A is a 'sufficient statistic' that encodes not only for the effect on demand of the prices of all other goods—domestic as well as imported—but also of income, I_A.

Other than producing different varieties, firms are identical within each country. Let $y_{i_A} \geq 0$ denote the quantity of variety i_A that firm i_A sells in the domestic market (i.e., in A), and let $x_{i_A} \geq 0$ denote the quantity of i_A that it sells abroad (i.e., in B). We say that a firm is active if $y_{i_A} + x_{i_A} > 0$.

Expressed in domestic labor units, firm i_A's cost function is

$$C_A[y_{i_A} + x_{i_A}] = F_A + c_A(y_{i_A} + x_{i_A}) \ .$$

Here, $F_A > 0$ denotes the fixed cost of operating (i.e., being active), which is sunk, and $c_A > 0$ denotes the constant marginal cost of production. As is customary in the literature, we work with 'productivity' $\phi_A \equiv 1/c_A \in (0, \infty)$, rather than with c_A itself. Similarly, we work with $\Phi_A \equiv 1/F_A \in (0, \infty)$ and refer to it as as fixed-cost 'efficiency.' To transform the labor cost $C_A[y_{i_A} + x_{i_A}]$ into monetary units, it must be multiplied by the domestic wage rate, $w_A > 0$.

The inverse-domestic-demand curve for variety i_A, $p_{i_A}[y_{i_A}, P_A]$, is found by aggregating the (binding) FOCs $P_A v'_A[z_{i_A}] = p_{i_A}$ in (2) over all households in A and using that demand must equal supply, y_{i_A}. Similarly, the inverse-foreign-demand curve $s_{i_A}[x_{i_A}, P_B]$ is found by aggregating the FOCs $P_B v'_B[z_{i_A}] = s_{i_A}$ over households in country B. This yields

$$p_{i_A}[y_{i_A}, P_A] = P_A v'_A[y_{i_A}/L_A] \text{ and } s_{i_A}[x_{i_A}, P_B] = P_B v'_B[x_{i_A}/L_B] \ . \tag{3}$$

Firm i_A's revenues from home-bound production y_{i_A} and exports x_{i_A} are

$$\mathcal{R}_A[y_{i_A}] \equiv P_A v'_A[y_{i_A}/L_A]\, y_{i_A} \text{ and } \mathcal{R}_B[x_{i_A}] \equiv P_B v'_B[x_{i_A}/L_B]\, x_{i_A} \ ,$$

respectively. Its profit, π_{i_A}, is

$$\pi_{i_A} = \mathcal{R}_A\left[y_{i_A}\right] + \mathcal{R}_B\left[x_{i_A}\right] - w_A C_A\left[y_{i_A} + x_{i_A}\right] \ .$$

Since firms are atomistic, individually, they do not influence wages w_A or price levels P_A, P_B. Hence, the FOC for optimal y_{i_A} is

$$FOC_{y_{i_A}} : \mathcal{R}'_A\left[y_{i_A}\right] \overset{(\leq)}{=} w_A/\phi_A \text{ if } y_{i_A} \overset{(=)}{>} 0 \ , \tag{4}$$

where marginal revenue, $\mathcal{R}'_A$, can be written as

$$\mathcal{R}'_A\left[y_{i_A}\right] = P_A v'_A\left[y_{i_A}/L_A\right]\left(1 - \varepsilon_{v'_A}\left[y_{i_A}/L_A\right]\right) \ . \tag{5}$$

We say that a solution to an FOC is interior, if the FOC holds with equality. (Thus, in (4), interiority does not necessarily preclude $y_{i_A} = 0$.) The FOC in (4) simply states that, at an interior optimum, marginal revenue equals marginal cost. The FOC $FOC_{x_{i_A}}$ for x_{i_A} and its interpretation are analogous. Observe that marginal revenue, $\mathcal{R}'_k[\cdot]$, $k \in \{A, B\}$, is strictly decreasing. Hence, the FOCs have at most one solution, and the SOC for a maximum is satisfied.

We denote firm i_A's optimal quantities by $\hat{y}_{i_A}\left[P_A/w_A\right]$ and $\hat{x}_{i_A}\left[P_B/w_A\right]$. Depending on the domestic wage w_A and price levels P_A, P_B, and conditional on being active, the firm either enters market $k \in \{A, B\}$ and produces the unique interior maximizer for that market, or it stays out and produces zero. Since optimal quantities are uniquely determined, all firms in country A behave identically, such that we need only keep track of the number of active firms, n_A, and not their identities, $i_A \in [0, n_A]$. With slight abuse of notation, we write y_A, p_A, x_A, s_A for y_{i_A}, p_{i_A}, x_{i_A}, s_{i_A}.

In the interior, (4) can be written as

$$\varepsilon_{v'_A}\left[\hat{y}_A/L_A\right] = \frac{p_A - w_A/\phi_A}{p_A} < 1 \ . \tag{6}$$

Since $\varepsilon_{v'_A}[0] = 0$, we have $0 \leq \varepsilon_{v'_A}\left[\hat{y}_A/L_A\right] < 1$—a property we rely on throughout. Similarly, $0 \leq \varepsilon_{v'_B}\left[\hat{x}_A/L_B\right] < 1$. Solving the equality in (6) for p_A yields

$$p_A = \frac{w_A/\phi_A}{1 - \varepsilon_{v'_A}\left[y_A/L_A\right]} \ . \tag{7}$$

Thus, in the interior, the optimal markup over marginal cost, denoted by m_A, is

$$m_A[\hat{y}_A] \equiv \frac{1}{1-\varepsilon_{v'_A}[\hat{y}_A/L_A]} - 1 = \frac{\varepsilon_{v'_A}[\hat{y}_A/L_A]}{1-\varepsilon_{v'_A}[\hat{y}_A/L_A]} \ . \tag{8}$$

Notice that $m_A[\cdot]$ is weakly (strictly) increasing at $\hat{y}_A \overset{(=)}{>} 0$. The expressions for s_A and $m_B[\hat{x}_A]$ are analogous and have the same properties.

Beyond paying fixed cost $1/\Phi_A > 0$, there are no barriers to becoming active, nor to ceasing activity. Hence, in equilibrium, the number of active firms, n_A, is such that the marginal firm makes zero profit. Because firms are symmetric within a country, this means that *all* firms make zero profit, i.e.,

$$\pi_A = \mathcal{R}_A[y_A] + \mathcal{R}_B[x_A] - w_A C_A[y_A + x_A] = 0 \ . \tag{9}$$

Notice that n_A and n_B do not directly enter into the zero-profit condition (9)—that is, the number of firms only affects π_A indirectly, via price levels and wages.

While households are the ultimate owners of firms, firms make no profit in equilibrium. Hence, household income, I_A, only consists of wages: $I_A = w_A$. Substituting this expression back into the budget constraint and using market clearing yields country A's budget balance equation

$$n_A \mathcal{R}_A[y_A] + n_B \mathcal{R}_B[x_B] = w_A L_A \ , \tag{10}$$

which equates total expenditure (LHS) with income (RHS).

Labor market clearing requires that

$$n_A C_A[y_A + x_A] = L_A \ . \tag{11}$$

Finally, to close the model, we impose balance of payments,

$$n_A \mathcal{R}_B[x_A] = n_B \mathcal{R}_A[x_B] \ . \tag{12}$$

This means that the value of country A's exports, $n_A \mathcal{R}_B[x_A]$, is equal to its imports, $n_B \mathcal{R}_A[x_B]$.

2.2 Equilibrium

Equilibrium consists of a tuple $(P_k, w_k, n_k)_{k\in\{A,B\}}$ of price indices P_k, wages w_k, and numbers of active firms n_k, inducing optimal quantities $\hat{y}_k[P_k, w_k]$, $\hat{x}_k[P_l, w_k]$, and prices $p_k[\hat{y}_k, P_k]$, $s_k[\hat{x}_k, P_l]$, $l \neq k$, such that zero profits (ZP_k, (9)), budget balance (BB_k, (10)), labor market clearing (LM_k, (11)), and balance of payments (BP, (12)) hold. In line with Walras' Law, one of these (pairs of) equations is redundant. To see this, substitute LM_k and BP into ZP_k to find BB_k. Equilibrium is thus characterized by the following system:

For $k, l \in \{A, B\}$, $l \neq k$,

$$\begin{aligned} ZP_k: \quad & \mathcal{R}_k[\hat{y}_k] + \mathcal{R}_l[\hat{x}_k] = w_k C_k[\hat{y}_k + \hat{x}_k] \\ LM_k: \quad & n_k C_k[\hat{y}_k + \hat{x}_k] = L_k \\ BP: \quad & n_A \mathcal{R}_B[\hat{x}_A] = n_B \mathcal{R}_A[\hat{x}_B] \ . \end{aligned} \tag{13}$$

This system contains five equations—BP and two each of ZP_k and LM_k—and six unknowns—P_k, w_k, n_k, $k \in \{A, B\}$. To solve the system, we normalize $w_A = 1$.

The next proposition, whose proof can be found in Morgan *et al.* (2023), establishes existence of equilibrium.[4]

Proposition 1 *Equilibrium exists and entails trade.*

Since $\hat{x}_k > 0$ in equilibrium, FOC_{x_k} holds with equality, $k \in \{A, B\}$. On the other hand, FOC_{y_k} may be slack, as $\hat{y}_k$ can be zero. Substituting the expression for p_k and s_k from (the analog of) (7) into ZP_k yields, after minor rewriting,

$$ZP_k: m_k[\hat{y}_k]\,\hat{y}_k + m_l[\hat{x}_k]\,\hat{x}_k = \phi_k/\Phi_k \ . \tag{14}$$

Observe that the equality holds even when FOC_{y_k} is slack, i.e., when $\hat{y}_k = 0$. We denote autarky values by a tittle, '˙'. The autarky value for home-bound production, $\dot{y}_k$, is found by setting $\hat{x}_k = 0$ in (14). Since $m_k[\cdot]$ is increasing and $m_k[0] = 0$, $\dot{y}_k$ is the unique solution to

$$m_k[\dot{y}_k]\,\dot{y}_k = \phi_k/\Phi_k \ . \tag{15}$$

3 When Autarky Trumps Free and Costless Trade

Krugman (1979) demonstrated that free and costless trade Pareto-dominates autarky. Here, we show that this is an artifact of assuming that countries are symmetric: if we relax this assumption, a country may be better off in autarky than in free and costless trade.

The remainder of this section is organized as follows. First, we present a simple example. Then we establish our main result, showing that our example is, in fact, generic. Finally, we develop an intuition. To simplify notation, we suppress the circumflex on $\hat{y}_k$ and $\hat{x}_k$.

[4] In a recent paper, Slepov and Kokovin (2023) prove equilibrium existence for more than two countries.

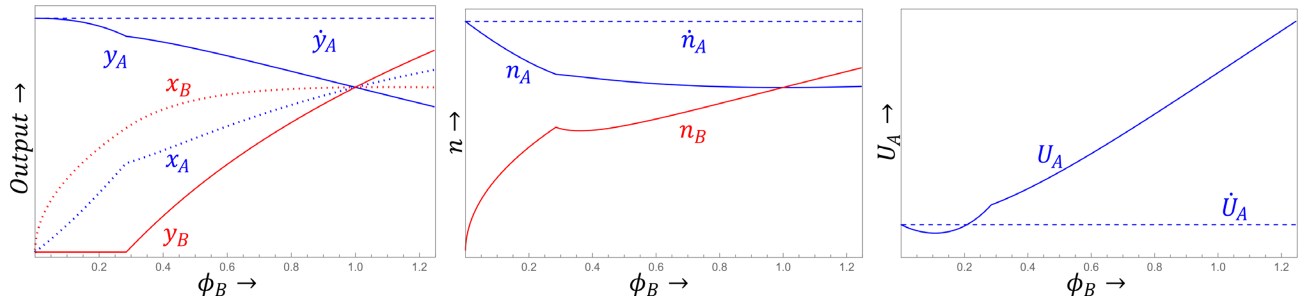

Figure 1: Example of Section 3.1. The figure depicts firm-level outputs (left panel), the number of firms (middle panel) and utility (right panel) as a function of productivity ϕ_B. Preferences are Pollak, $v[z] = (z+\gamma)^{\rho} - \gamma^{\rho}$, with $\gamma > 0$.

3.1 Example

Countries A and B engage in free and costless trade, as described in Section 2. Except for their productivities, the two countries are symmetric. Keeping productivity in country A fixed, we trace out equilibrium as ϕ_B increases from zero in the limit to a value somewhat greater than ϕ_A. Plotting U_A, we find that utility of households in A is U-shaped in ϕ_B. In particular, U_A is downward sloping at $\phi_B = 0$. Observe that, all along this curve, trade is free and costless. However, at $\phi_B = 0$, the two countries also live in autarky, for the simple reason that country B no longer produces anything. Slightly increasing ϕ_B from zero ends autarky but maintains free and costless trade. Since U_A falls, this means that, in a right-neighborhood of $\phi_B = 0$, country A is strictly better off in autarky than in free and costless trade.

More specifically, to generate Figure 1, we let $v_A[z] = v_B[z] = (\gamma + z)^{\frac{1}{2}} - \gamma^{\frac{1}{2}}$, $\gamma = 10$. These so-called Pollak preferences have finite choke prices if $\gamma > 0$ and reduce to CES for $\gamma = 0$. Country A has productivity $\phi_A = 1$, while country B's productivity, ϕ_B, varies from just above 1 to 0. Otherwise, the two countries are symmetric and parameterized by $\Phi_A = \Phi_B = 10^{-4}$ and $L_A = L_B = 10^6$.

As a function of ϕ_B, Figure 1 depicts: *(i)* per-firm output for the home market, y_A, y_B—solid lines, left panel; *(ii)* per-firm exports, x_A, x_B—dotted lines, left panel; *(iii)* the number of firms, i.e., varieties, n_A, n_B—middle panel; and *(iv)* utility U_A—right panel. U_B, which has been omitted, equals U_A for $\phi_B = 1$ and, as ϕ_B falls, drops fast toward zero, visually dwarfing changes in U_A. Autarky production $\dot{y}_A$, number of firms $\dot{n}_A$, and utility $\dot{U}_A$ are shown for reference by dashed lines.

When $\phi_B = 1 = \phi_A$, countries are symmetric, and their equilibrium quantities and utilities are identical. Furthermore, $y_k = x_k$, $k \in \{A, B\}$, owing to trade being free and costless. As productivity ϕ_B decreases, per-firm exports x_A, x_B fall in both countries, as does home-bound production y_B in the less productive country, B. Since y_B and x_B both decrease, so does the scale of production, $y_B + x_B$—a logical response to lower productivity. Scale $y_A + x_A$ in country A also contracts, despite y_A going up. This is line with the common finding that a fall (rise) in trade reduces (increases) scale.[5] For A, a reduction in scale with unchanged costs implies that the number of firms, n_A, unambiguously rises. By contrast, as ϕ_B drops, n_B tends to decrease. This means that, for B, the direct effect of a decrease in its productivity—namely, greater use of labor—dominates the indirect effect—namely, the reduction in scale associated with a fall in trade.

In B, home-bound production y_B drops to zero at $\phi_B = 0.3$. That is, for $\phi_B \leq 0.3$, domestic varieties are so expensive that households in B stop consuming them; they are for export only. As ϕ_B falls further, x_B and n_B continue to decrease, until $x_B = n_B = 0$ in the limit for $\phi_B \to 0$. At that point, country B produces zero output and both countries live in autarky.

Finally, and most importantly, notice that U_A is U-shaped in ϕ_B, converging to autarky utility $\dot{U}_A$ as $\phi_B \to 0$. As a result, in this example, country A is strictly better off in autarky than in free and costless trade for all $\phi_B \in (0, 0.2)$. Starting from $\phi_B = 0.5$, say, and decreasing, U_A drops below $\dot{U}_A$ at $\phi_B = 0.2$ and, as ϕ_B continues to fall, utility never fully recovers, except in the limit.

In the next section, we show that U-shapedness of U_A in ϕ_B is a general property in the Krugman model. Together with convergence to autarky for $\phi_B \to 0$, this implies our central claim, namely, that autarky trumps free and costless trade when facing a low-productivity trading partner.

3.2 The Result

For concreteness, we take again the perspective of country A. Fixing model parameters other than $\phi_B \in (0, \infty)$, let $U_A[\phi_B]$ denote utility in A as a function of productivity in B. The following theorem summarizes the main result of the paper.

Theorem 1 (Autarky Trumps Free Trade) *A country is strictly better off in autarky than in free and costless trade, when the productivity of its trading partner is sufficiently low.*

Formally, $\dot{U}_A > U_A[\phi_B]$ *in a neighborhood of* $\phi_B = 0$*.*

The proof of Theorem 1, which has been relegated to the Appendix, consists of two parts. In the first part, we show that $\lim_{\phi_B \to 0} U_A[\phi_B] = \dot{U}_A$. That is, when country B becomes wholly unproductive, countries end up in autarky. This is intuitive and easy to prove. Since all equilibria

[5] The positive relationship between trade and scale (firm size), relies on increasing $\varepsilon_{v'}$, which we, along with Krugman (1979), have assumed. Zhelobodko *et al.* show that $\varepsilon_{v'}$ increasing is, in fact, the 'normal' case, giving rise to intuitive, pro-competitive effects of market entry, namely, lower prices and mark-ups. Conversely, decreasing $\varepsilon_{v'}$ has the opposite effect. Under symmetric CES preferences, firm size is famously invariant to trade (see, e.g., Krugman, 1980, footnote 3).

converge to this point, the implicit function theorem implies that equilibrium is unique in a neighborhood around autarky. In the second part, which is more involved, we implicitly differentiate the equilibrium system and show that $\lim_{\phi_B \to 0} dU_A/d\phi_B < 0$. Certain parts of this derivative go to zero or blow up as $\phi_B \to 0$. Therefore, calculating the limit requires some careful collecting and parsing of factors and terms. The insights yielded by these algebraic manipulations is, at times, limited. Instead of going through the proof line-by-line, in the main text we develop a, partially graphical, intuition for Theorem 1. To help the reader link the intuition to the proof, we reference the relevant lemmas in the Appendix at appropriate times in the argument.

3.3 An Intuition

In this section, we develop an intuition for Theorem 1. In the limit for $\phi_B \to 0$, country A lives in autarky *and* in free and costless trade. The latter claim is trivial, because our model has neither tariffs nor trade costs. To see why A ends up in autarky, observe that B effectively ceases production as $\phi_B \to 0$. Since B can no longer supply any goods to A, trade ceases. Consequently, $x_A, x_B \to 0$, $y_A \to \dot{y}_A$, and $\lim_{\phi_B \to 0} U_A = \dot{U}_A$. (See Lemmas 3 to 6 in the Appendix for proofs of these claims.)

Starting from the limit and reversing course, a marginal increase in ϕ_B initiates trade. Importantly, in the new equilibrium, y_A, p_A, and P_A retain their original autarky values, at least to the first order, while the number of domestic varieties, n_A, falls. To see why, implicitly differentiate ZP_A in (14) with respect to ϕ_B, yielding

$$\left(m_A\left[y_A\right] + m'_A\left[y_A\right] y_A\right) \frac{dy_A}{d\phi_B} + \left(m_B\left[x_A\right] + m'_B\left[x_A\right] x_A\right) \frac{dx_A}{d\phi_B} = 0 \; . \tag{16}$$

Lemma 12 confirms that $dx_A/d\phi_B$ remains finite as $\phi_B \to 0$. For future reference, we note that this property critically depends on finite choke prices, or $v'[0] < \infty$.[6] Since $\lim_{\phi_B \to 0} y_A = \dot{y}_A$ and $\lim_{\phi_B \to 0} x_A = 0 = m_B[0]$, we find that

$$\left(m_A\left[\dot{y}_A\right] + m'_A\left[\dot{y}_A\right] \dot{y}_A\right) \lim_{\phi_B \to 0} \frac{dy_A}{d\phi_B} = 0 \; . \tag{17}$$

Hence, $\lim_{\phi_B \to 0} dy_A/d\phi_B = 0$. In turn, the constancy of y_A implies that P_A remains unchanged as well, i.e., $\lim_{\phi_B \to 0} dP_A/d\phi_B = 0$. This follows from FOC_{y_A} in (4) with $w_A = 1$. Finally, p_A stays constant because $p_A = P_A v'_A\left[y_A/L_A\right]$.

[6] Specifically,

$$\lim_{\phi_B \to 0} \frac{dx_A}{d\phi_B} = \lim_{\phi_B \to 0} \frac{x_A}{\phi_B} = \frac{L_B}{L_A} \frac{v'_A[0]\, C_A\left[\dot{y}_A\right]}{v'_A\left[\dot{y}_A/L_A\right]} \left(1 + m_A\left[\dot{y}_A\right]\right) < \infty \; ,$$

where the inequality follows from $v'_A[0] < \infty$. See Lemmas 9 and 12.

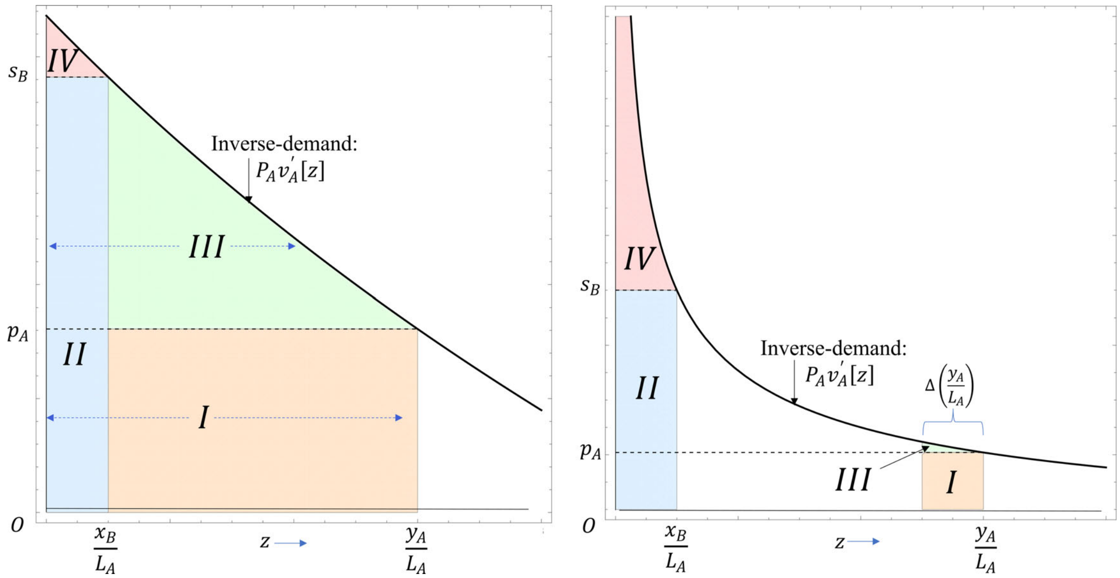

Figure 2: Demand and consumer surplus with finite choke price (left panel) and infinite choke price (right panel).

Although y_A remains unchanged, the scale of production in country A, $y_A + x_A$, increases, because now $x_A > 0$. An increase in scale reduces the number of firms, because $n_A = L_A / C_A [y_A + x_A]$, which follows from LM_A in (13). Thus, the adjustment in domestic goods consumption in A, following a small increase in ϕ_B from zero, occurs entirely through a reduction in the number of domestic varieties, n_A, while the quantity per variety, y_A, remains unchanged.

Having have lost access to some domestic varieties, in the new equilibrium, households in A consume small amounts of a few foreign varieties. Otherwise, households' circumstances have remained unchanged. To see why exchanging domestic varieties *in their entirety* for marginal units of foreign varieties is disadvantageous, consider the left panel of Figure 2. Areas I and II represent household expenditure on a domestic and a foreign variety, respectively, while areas III and IV correspond to the associated consumer surpluses. (In the left panel, notice that areas I and II partially overlap, while area III fully contains IV.) Since choke prices are finite, the demand curve intersects the price axis, making surplus 'triangular.' As $\phi_B \to 0$ and $x_B \to 0$, surplus (IV) shrinks quadratically, while expenditure (II) contracts linearly.[7] Therefore, the surplus per dollar

[7] To see that surplus, IV, shrinks quadratically, notice that both the base and the height of the triangle are linear in x_B. To see that expenditure, II, shrinks linearly in x_B as $x_B \to 0$, observe that

$$\frac{d\,[s_B x_B / L_A]}{d x_B} = \frac{P_A v'\,[x_B/L_A]}{L_A} \left(1 - \varepsilon_{v_A'}\,[x_B/L_A]\right) \overset{x_B \to 0}{\longrightarrow} \frac{P_A v'\,[0]}{L_A} \in (0, \infty) \ .$$

(SPD) derived from a foreign variety, IV/II, vanishes. Meanwhile, as $y_A \to \dot{y}_A > 0$, the SPD from a domestic variety, III/I, remains strictly positive. Equivalently, this means that starting from the limit and marginally increasing ϕ_B, the surplus gained from a new foreign variety is second order, whereas the surplus lost from a displaced domestic variety is first order. Even though new foreign varieties outnumber lost domestic ones, the few first-order losses dominate the sum of many second-order gains.

A somewhat more formal analysis is as follows. Since the initiation of trade leaves the price level P_A unchanged, general and partial equilibrium effects coincide for country A in terms of utility. Thus, the displacement of domestic varieties by foreign ones makes households in A worse off iff it lowers the average SPD, i.e., iff

$$\frac{IV}{II} < \frac{III}{I} \iff \frac{II}{II+IV} > \frac{I}{I+III} \ . \tag{18}$$

The ratios $II/(II+IV)$ and $I/(I+III)$ on the RHS correspond to the 'cost per util' (CPU) of foreign and domestic varieties, respectively. The inverse-demand curve is given by $P_A v'_A[z]$. Therefore,

$$II = s_B x_B/L_A = P_A v'_A[x_B/L_A]\, x_B/L_A \text{ and } II+IV = \int_0^{\frac{x_B}{L_A}} P_A v'_A[z]\, dz = P_A v[x_B/L_A] \ . \tag{19}$$

The expressions for I and $I+III$ are analogous. Substituting these into the RHS of (18) yields

$$\frac{v'_A[x_B/L_A]\, x_B/L_A}{v[x_B/L_A]} > \frac{v'[y_A/L_A]\, y_A/L_A}{v_A[y_A/L_A]} \iff \varepsilon_{v_A}[x_B/L_A] > \varepsilon_{v_A}[y_A/L_A] \, . \tag{20}$$

Recall that $\lim_{\phi_B \to 0} y_A = \dot{y}_A > 0$ and $\lim_{\phi_B \to 0} x_B = 0$, while we know from Lemma 1 that $\varepsilon_{v_A}[z]$ is strictly decreasing. Hence, the inequality in (20) is satisfied in the limit, explaining why the initiation of trade leaves country A strictly worse off.

Notably, the inequality in (20) is identical to the condition found by Venables (1982) for when the displacement of domestic varieties by foreign ones is disadvantageous. While Venables considered an, essentially, partial equilibrium environment, here the condition extends to general equilibrium, because small changes in ϕ_B near zero do not affect domestic prices and the price level, p_A and P_A.

The proof of Theorem 1 in the Appendix adopts more of 'brute-force' approach. Differentiating $U_A = n_A v_A[y_A/L_A] + n_B v_A[x_B/L_A]$ with respect to ϕ_B yields

$$\frac{dU_A}{d\phi_B} = v_A[y_A/L_A]\frac{dn_A}{d\phi_B} + n_A v'_A[y_A/L_A]\frac{1}{L_A}\frac{dy_A}{d\phi_B} + \frac{d}{d\phi_B}[n_B v_A[x_B/L_A]] \ .$$

Letting $\phi_B \to 0$,

$$\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} = v_A\left[\dot{y}_A/L_A\right] \lim_{\phi_B \to 0} \frac{dn_A}{d\phi_B} + \lim_{\phi_B \to 0} \frac{d}{d\phi_B}\left[n_B v_A\left[x_B/L_A\right]\right] \ . \tag{21}$$

Here we have used that $\lim_{\phi_B \to 0} y_A = \dot{y}_A$ and $\lim_{\phi_B \to 0} dy_A/d\phi_B = 0$. In Lemma 18 it is shown that

$$\begin{aligned}
\lim_{\phi_B \to 0} \frac{d}{d\phi_B}\left[n_B v_A\left[x_B/L_A\right]\right] &= \lim_{\phi_B \to 0} v_A\left[x_B/L_A\right] \frac{dn_B}{d\phi_B} + \lim_{\phi_B \to 0} n_B v_A'\left[x_B/L_A\right] \frac{1}{L_A}\frac{dx_B}{d\phi_B} \\
&= \frac{1}{2}\frac{L_B}{L_A} v_A'\left[0\right] + \frac{1}{2}\frac{L_B}{L_A} v_A'\left[0\right] = \frac{L_B}{L_A} v_A'\left[0\right] \ .
\end{aligned} \tag{22}$$

Observe that these derivatives are non-zero, despite $\lim_{\phi_B \to 0} n_B = \lim_{\phi_B \to 0} x_B = 0$, because $\lim_{\phi_B \to 0} dn_B/d\phi_B = \lim_{\phi_B \to 0} dx_B/d\phi_B = \infty$. Lemma 16 establishes that

$$\lim_{\phi_B \to 0} \frac{dn_A}{d\phi_B} = -\frac{L_B}{\dot{y}_A} \frac{v_A'\left[0\right]}{v_A'\left[\dot{y}_A/L_A\right]} \ . \tag{23}$$

Substituting (22) and (23) into (21) and rearranging,

$$\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} = -\frac{L_B}{L_A} \frac{v_A'\left[0\right]}{\varepsilon_{v_A}\left[\dot{y}_A/L_A\right]} + \frac{L_B}{L_A} v_A'\left[0\right] = \left(1 - \frac{1}{\varepsilon_{v_A}\left[\dot{y}_A/L_A\right]}\right) \frac{L_B}{L_A} v_A'\left[0\right] < 0 \ , \tag{24}$$

where the inequality follows from $\varepsilon_{v_A}\left[\dot{y}_A/L_A\right] < 1$ (Lemma 1). Equation (24) reveals that the 'Bad Trade' effect is locally increasing in L_B/L_B, $\dot{y}_A$, and $v_A'\left[0\right]$.[8] The larger country B is relative to A, the more its exports expand in response to a marginal increase in ϕ_B, amplifying the negative impact on A. Regarding $\dot{y}_A$, recall that consumer surplus grows quadratically, while expenditure increases linearly. Hence, the SPD rises with $\dot{y}_A$, causing a sharper utility decline when foreign varieties displace domestic ones. Finally, $v_A'\left[0\right]$ primarily serves as a scaling factor.

While switching to autarky benefits country A, it harms country B. The reason is that, for small ϕ_B, home-bound production y_B of firms in B *equals* zero, rather than gradually approaching it as do x_A and x_B (see Lemma 7 and Figure 1). Households in B cease consuming domestically produced varieties, because they have become too expensive relative to foreign ones. In the absence of infra-marginal units of y_B, a switch to autarky forces country B to forego the somewhat-positive-surplus units x_A for, on the margin, zero-surplus units y_B. Consequently, country B loses from autarky.

[8] While $\dot{y}_A$ is endogenous, it varies monotonically with ϕ_A/Φ_A, which is exogenous.

4 Why CES is Different

One may wonder why 'Bad Trade' in the Krugman model has remained unnoticed for so long. Compared to Krugman (1979), Krugman (1980) introduced an additional assumption, namely, symmetric CES utility $v[z] = z^{\rho}$, where $0 < \rho < 1$. This assumption allowed for closed-form solutions and became integral to most New Trade and 'New' New Trade models. However, CES also has the singular property of infinite choke prices, or $v'[0] = \infty$.

Finite choke prices are critical to the proof of Theorem 1. Recall that $\lim_{\phi_B \to 0} dy_A/d\phi_B = 0$ hinges on $v'[0] < \infty$. With CES, by contrast, $v'[0] = \infty$, and $\varepsilon_{v'}[0] = \varepsilon_{v'}[z] = 1 - \rho > 0$ for $z \in [0, \infty)$. In that case, ZP_A simplifies to

$$ZP_A : \frac{1-\rho}{\rho}(y_A + x_A) = \frac{\phi_A}{\Phi_A} \ .$$

Implicitly differentiating with respect to ϕ_B, we obtain

$$\frac{d}{d\phi_B}[y_A + x_A] = 0 \ .$$

Since $\lim_{\phi_B \to 0} dx_A/d\phi_B > 0$, it follows that $\lim_{\phi_B \to 0} dy_A/d\phi_B < 0$. Additionally,

$$\frac{dn_A}{d\phi_B} = \frac{d}{d\phi_B}\left[\frac{L_A}{C_A[y_A + x_A]}\right] = 0$$

holds for all ϕ_B, not just in the limit. I.e., the number of varieties in A is constant in ϕ_B. With finite choke prices, recall that ϕ_B rising above zero caused n_A to adjust, while y_A remained constant. With symmetric CES, by contrast, the opposite occurs: y_A adjusts while n_A remains constant. Consequently, the trade-off central to Theorem 1—the exchange of domestic varieties as a whole for marginal units of new foreign varieties—no longer applies. Instead, the trade-off shifts to one between marginal units of domestic varieties and marginal units of new foreign varieties, keeping the number of domestic varieties constant. We now show that this turns Theorem 1 on its head.

Proposition 2 *Suppose utility is symmetric CES. Country A is strictly better off in free and costless trade than in autarky, when its trading partner's productivity ϕ_B is sufficiently low.*

Formally, $\dot{U}_A < U_A[\phi_B]$ in a neighborhood of ϕ_B around zero.

To develop an intuition for Proposition 2, consider the right panel of Figure 2. Area I represents the cost of the 'last' unit, $\Delta(y_A/L_A)$, of a domestic variety, while III corresponds to the surplus derived from it. Similarly, area II represents the expenditure on the 'first' unit, x_B/L_A, of a foreign variety, with IV corresponding to its surplus. Now, the CPUs are

$$\frac{II}{II+IV} = \frac{v'[x_B/L_A]\, x_B/L_A}{v[x_B/L_A]} = \rho < 1 \ ,$$

and

$$\frac{I}{I+III} = \frac{v'_A[z] \cdot \Delta(y_A/L_A)}{v[y_A/L_A] - v[y_A/L_A - \Delta(y_A/L_A)]} \to 1 \text{ as } \Delta(y_A/L_A) \to 0 \ .$$

Hence, $II/(II+IV) < I/(I+III)$, meaning that the exchange of 'last' units of domestic varieties for 'first' units of foreign ones reduces the CPU—or, equivalently, raises the SPD—benefiting households in A.

However, this is only the partial equilibrium (PE) side of the story. With finite choke prices, P_A remained constant when ϕ_B rose above zero, making GE equal to PE. With CES, by contrast, P_A declines, further benefiting households in A. This follows from y_A falling and FOC_{y_A} in (4). In fact, as $\lim_{\phi_B \to 0} dy_A/d\phi_B = -\infty$ (Lemma 20), the decline in the price level is locally unbounded. As a result, $dU_A/d\phi_B$ is not merely positive but infinite in the limit (see Lemma 21).[9]

We suspect that Proposition 2 extends to asymmetric CES preferences, $v_k[z] = z^{\rho_k}$, $0 < \rho_k < 1$, $k \in \{A, B\}$. However, we do not have a proof. Asymmetric CES lacks the analytical tractability of both symmetric CES and of finite choke prices. Under symmetric CES, $dU_A/d\phi_B$ can be computed in closed form and its limit is easily established (see Lemma 21). With $v'[0] < \infty$, signing $\lim_{\phi_B \to 0} dU_A/d\phi_B$ is simplified by the fact that y_B is constant and equal to zero for ϕ_B sufficiently small. In contrast, asymmetric CES offers neither advantage. Signing $\lim_{\phi_B \to 0} dU_A/d\phi_B$ then requires implicitly differentiating the full system of equilibrium equations, solving for the derivatives, substituting them into $dU_A/d\phi_B$, and taking the limit as $\phi_B \to 0$. It also requires determining the relative rates at which x_A, y_B, x_B, and n_B approach zero and trading them off against each other. We have not succeeded in doing that.

In the absence of a proof for the converse of Theorem 1, the following example demonstrates the weaker result that Theorem 1 does not extend to asymmetric CES.

Example 2 *Preferences are CES, $v_k[z] = z^{\rho_k}$. The upper panels of Figure 3 depict per-firm output and utility in country A as a function of ϕ_B for $\rho_A = 0.5 < \rho_B = 0.6$. Analogous plots for the cases $\rho_A = \rho_B$ and $\rho_A > \rho_B$ are qualitatively similar and, therefore, omitted. The lower panels depict the number of firms (varieties) as a function of ϕ_B for $\rho_A = 0.5 < \rho_B = 0.6$, $\rho_A = \rho_B = 0.5$, and $\rho_A = 0.6 > \rho_B = 0.5$, respectively. All other model parameters match the main example in Section 3.1.*

[9] Under CES, constant markup over marginal cost implies that p_A is invariant in ϕ_B. This also follows from FOC_{y_A}.

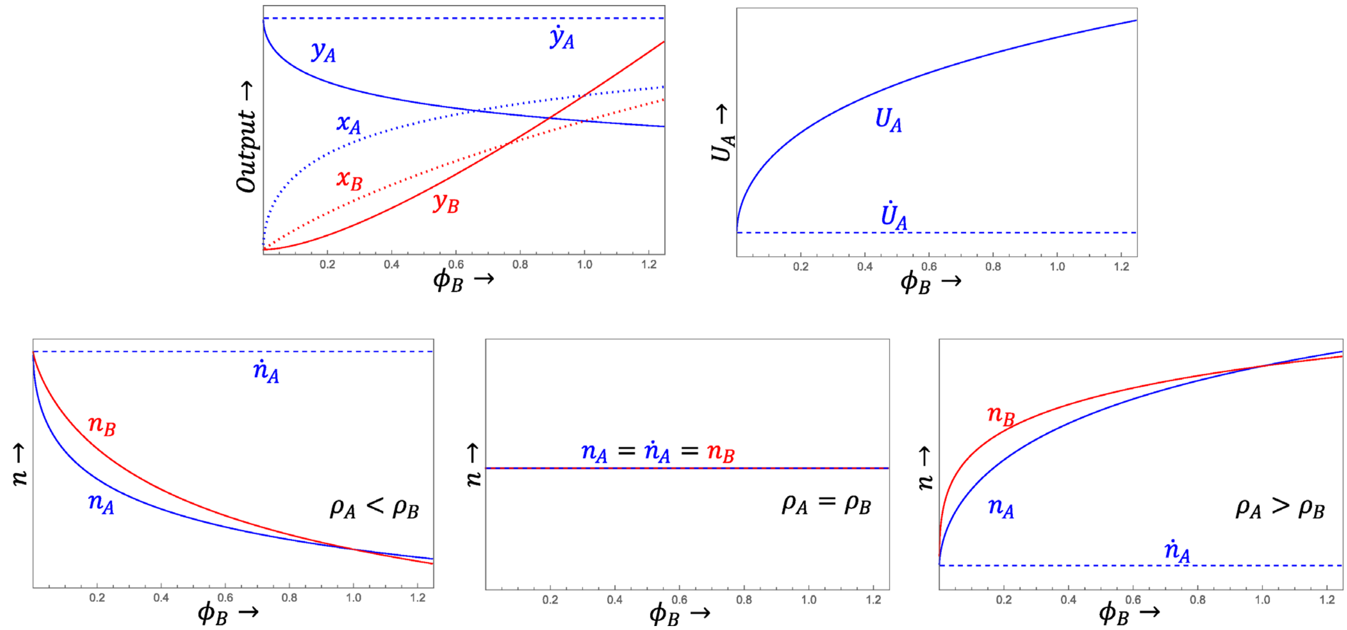

Figure 3: Example 2. CES preferences, $v_k[z] = z^{\rho_k}$. Top row: Output and U_A for $\rho_A < \rho_B$. For $\rho_A = \rho_B$ and $\rho_A > \rho_B$, the figures are very similar and have been omitted. Bottom row: number of firms (varieties) n with $\rho_A < \rho_B$, $\rho_A = \rho_B$, and $\rho_A > \rho_B$, respectively.

When $\phi_B = 0$, both countries live in autarky. As ϕ_B rises above zero, trade begins and x_B increases. However, unlike with finite choke prices, the quantity of each domestic variety in A, y_A, declines sharply. This steep decline of y_A is accompanied by a similar drop in the price level P_A (not depicted), which sharply boosts welfare in A. When $\rho_A < \rho_B$—that is, when households in A have a stronger taste for variety than those in B—both n_A and n_B fall. This contrasts with the baseline model, in which n_A decreases while n_B rises from zero. When $\rho_A = \rho_B$ (so that Proposition 2 applies), output and utility in A evolve as before with increasing ϕ_B, but scale and variety remain constant, i.e. $d[y_A + x_A]/d\phi_B = 0 = d[y_B + x_B]/d\phi_B$ (not depicted) and $n_A = \dot{n}_A = n_B$. Finally, when $\rho_A > \rho_B$, both n_A and n_B rise with ϕ_B.

With asymmetric CES, neither n_A nor y_A remain fixed when ϕ_B exceeds zero—unlike with symmetric CES or finite choke prices, respectively. Moreover, since n_A and n_B move in the *same* direction—rising or falling depending on whether $\rho_A \lesseqgtr \rho_A$—the impact of increasing ϕ_B cannot be reduced to a straightforward combination of the effects observed under symmetric CES and finite choke prices.

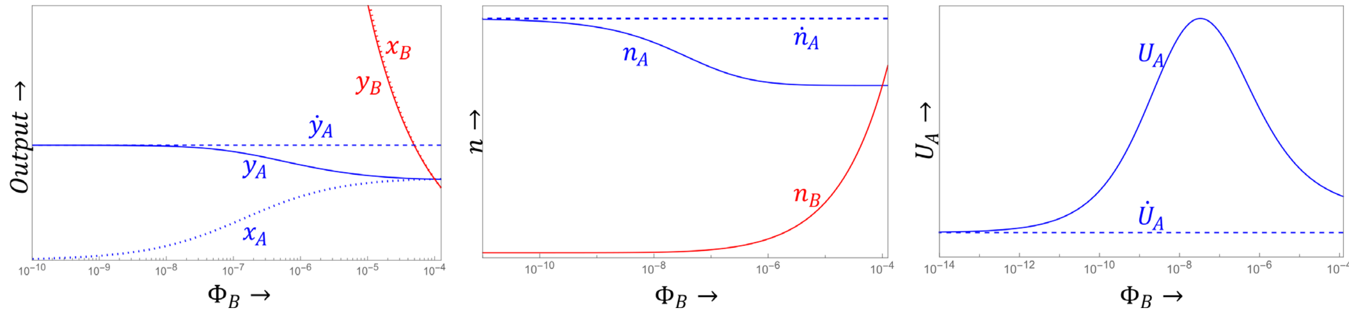

Figure 4: Example 3. Firm-level outputs (left panel), the number of firms (middle panel) and utility (right panel) as a function of fixed-cost efficiency Φ_B. Preferences are as in the main example in Section 3.1.

5 Letting Φ_B go to Zero

Returning to our baseline model with finite choke prices, one might expect that country A also fares better in autarky than under free and costless trade if fixed-cost efficiency Φ_B becomes sufficiently small. This is because, in the limit, country A once again lives in autarky and in free and costless trade simultaneously. Furthermore, a small increase in Φ_B from zero initiates trade. (These assertions are intuitive and formally proved in Appendix B.) Despite the apparent similarity with $\phi_B \to 0$, Theorem 1 does not extend to this scenario. That is, country A may benefit from an increase in Φ_B.

Example 3 *Figure 4 depicts per-firm output, the number of firms, and utility as a function of fixed-cost efficiency, Φ_B. The horizontal axis is in log-scale to facilitate visualization around $\Phi_B = 0$. Other parameters match the main example in Section 3.1.*

As $\Phi_B \to 0$, country B's per-firm exports, x_B, surge. Still, country A lives in autarky in the limit, as n_B declines even faster, ensuring that $n_B x_B \to 0$. Unlike in the main example in Section 3.1, but similar to CES, y_A and P_A (not shown) decreases rapidly in linear scale, while U_A increases. The number of domestic varieties n_A decline, whereas foreign varieties n_B expand.

We conjecture that Example 3 generalizes, in the sense that country A always benefits from small increases in Φ_B near zero. However, as with asymmetric CES and $\phi_B \to 0$, we lack a formal proof. Our argument is as follows. While low productivity ϕ_B discourages scale in B, low efficiency Φ_B encourages it—an intuitive outcome. Specifically,

$$\lim_{\Phi_B \to 0} x_B = \mathcal{R}_A^{-1}[0] / P_A = \varepsilon_{v'_A}^{-1}[1] > \dot{y}_A \; . \tag{25}$$

(See Lemma 27. With Pollak preferences, $\varepsilon_{v'_A}^{-1}[1] = \infty$.) Example 3 suggests that as Φ_B increases, only n_B and x_A rise, while x_B, n_A, and y_A decline, along with p_A and P_A, which follows from FOC_{y_A}. These patterns suggest that a marginal rise in Φ_B affects A's households as follows: (1) As y_A falls, households reallocate spending away from 'last' units of domestic varieties. In PE, this is, at worst, welfare neutral since the SPD of these units is zero. In GE, the associated drop in price p_A and price level P_A strictly benefits A. (2) Although x_B declines, in the limit, no money is spent on foreign varieties, since $\lim_{\Phi_B \to 0} p_B n_B x_B = 0$. So, the apparent 'loss' of x_B does not harm A. (3) Since $\lim_{\Phi_B \to 0} x_B > 0$, households now exchange domestic varieties as a whole for foreign varieties as a whole. Revisiting condition (20), this is beneficial because

$$\varepsilon_{v_A}\left[\varepsilon_{v'_A}^{-1}[1]\right] < \varepsilon_{v_A}[\dot{y}_A / L_A] \ ,$$

which follows from decreasingness of $\varepsilon_v[\cdot]$ (Lemma 1) and (25). Overall, these effects suggest that A benefits when Φ_B rises above zero.

As shown in Appendix C, Theorem 1 does not extend to small L_B either. That is, country A may benefit when L_B rises above zero. However, we remain agnostic about the generality of this result. First, we cannot rule out that there exist equilibria where A converges to a state other than autarky as $L_B \to 0$. Second, even if A converges to autarky, the welfare effect of increasing L_B may be ambiguous. Nevertheless, we have yet to find an example where autarky trumps free and costless trade for small L_B.

6 Conclusion

Krugman's (1979) seminal model laid the groundwork for New Trade Theory. By relaxing the assumption of symmetry between countries, we have derived a sufficient condition for a country to be better off in autarky than under free and costless trade. This result critically relies on finite choke prices and, as such, does not apply to CES preferences.

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A Proofs

Proof of Lemma 1:

Proof We suppress the country subscript. Non-negativity of ε_v follows from its definition and the properties of $v[\cdot]$.

Recall that $\varepsilon_{v'}[\cdot]$ is strictly increasing in a neighborhood of $z = 0$ and non-decreasing everywhere else. Therefore,

$$\begin{aligned}
\varepsilon_{v'}[z]\, v[z] &= \frac{-zv''[z]}{v'[z]} v[z] = \frac{-zv''[z]}{v'[z]} \int_0^z v'[\zeta]\, d\zeta > \int_0^z \frac{-\zeta v''[\zeta]}{v'[\zeta]} v'[\zeta]\, d\zeta \\
&= -\int_0^z \zeta v''[\zeta]\, d\zeta = -\left(\zeta v'[\zeta]\big|_0^z - \int_0^z v'[\zeta]\, d\zeta \right) = v[z] - zv'[z],
\end{aligned}$$

where we have used that $v[0] = 0$. Now notice that

$$\varepsilon_{v'}[z]\, v[z] = \frac{-zv''[z]}{v'[z]} v[z] > v[z] - zv'[z] \iff \left(v'[z] + zv''[z]\right) v[z] - zv'[z]^2 < 0 \;, \tag{26}$$

while differentiating $\varepsilon_v[z]$ and using (26) yields

$$\frac{d}{dz} \frac{zv'[z]}{v[z]} = \frac{(v'[z] + zv''[z])\, v[z] - zv'[z]^2}{v[z]^2} < 0 \;.$$

This proves that $\varepsilon_v[\cdot]$ is decreasing.

Finally, using Hopital's rule,

$$\lim_{z\to 0} \varepsilon_v[z] = \lim_{z\to 0} \frac{v'[z]\, z}{v[z]} = \lim_{z\to 0} \frac{v'[z] + v''[z]\, z}{v'[z]} = 1 \;.$$

■

A.1 Proof of Theorem 1

To prove Theorem 1, we show that $\lim_{\phi_B \to 0} U_A = \dot{U}_A$ and $\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} < 0$. Jointly, the two results imply the proposition. Concretely, the proof proceeds in five steps.

1. Rewrite the equilibrium system solely in terms of y_k, x_k, $k \in \{A, B\}$, and the wage ratio w_A/w_B.

2. Derive the limits for y_k, x_k, n_k as $\phi_B \to 0$, and show that $\lim_{\phi_B \to 0} U_A = \dot{U}_A$.

3. Show that the Implicit Function Theorem (IFT) applies for all $\phi_B > 0$ sufficiently small.

4. Derive the limits of the derivatives of y_k, x_k, n_k with respect to ϕ_B, as $\phi_B \to 0$.

5. Use these limit values to demonstrate that $\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} < 0$.

A.1.1 Rewriting the Equilibrium System

Let $R_k[z] \equiv \mathcal{R}_k[z]/P_k = v_k'[z/L_k]\,z$ denote price-level-normalized gross revenue, and observe that

$$R_k'[z] \equiv \frac{\mathcal{R}_k'[z]}{P_k} = v_k'[z/L_k]\left(1 - \varepsilon_{v_k'}[z/L_k]\right) .$$

The next lemma permits focusing on a self-contained sub-system of equations that is only a function of production quantities and the wage ratio.

Lemma 2 *In equilibrium,*

$$\begin{aligned} &FOC^k : \frac{R_k'[y_k]}{R_k'[x_l]} \overset{(\leq)}{=} \frac{w_k/\phi_k}{w_l/\phi_l} \quad \text{if } y_k \overset{(=)}{>} 0 \\ &ZP^k : m_k[y_k]\,y_k + m_l[x_k]\,x_k = \phi_k/\Phi_k \\ &BP : \frac{L_l}{L_k}\frac{C_k[y_k+x_k]}{C_l[y_l+x_l]}\frac{1+m_k[x_l]}{1+m_l[x_k]}\frac{x_l}{x_k} = \frac{w_k/\phi_k}{w_l/\phi_l} , \end{aligned} \tag{27}$$

for $k,l \in \{A,B\}$, $l \neq k$. The five unknowns are y_A, x_A, y_B, x_B, and w_B/w_A.

Proof Since countries trade for all $\phi_B \in (0,\infty)$, we have $x_A, x_B > 0$. Hence, FOC_x^k in (the analog of) (4) must be binding, $k \in \{A,B\}$. However, FOC_y^k may be slack. In that case, $y_k = 0$, and firms in country k produce only for the export market. Equilibrium is then characterized by:

$$\begin{aligned} &FOC_y^k : \quad P_k v_k'[y_k/L_k]\left(1-\varepsilon_{v_k'}[y_k/L_k]\right) \overset{(\leq)}{=} w_k/\phi_k \quad \text{if} \quad y_k \overset{(=)}{>} 0 \\ &FOC_x^k : \quad P_l v_l'[x_k/L_l]\left(1-\varepsilon_{v_l'}[x_k/L_l]\right) = w_k/\phi_k \\ &ZP^k : \quad P_k v_k'[y_k/L_k]\,y_k + (1-r_l)\,P_l v_l'[x_k/L_l]\,x_k = w_k C_k[y_k+x_k] \\ &LM^k : \quad n_k C_k[y_k+x_k] = L_k \\ &BP : \quad n_k(1-r_l)\,P_l v_l'[x_k/L_l]\,x_k = n_l(1-r_k)\,P_k v_k'[x_l/L_k]\,x_l , \end{aligned} \tag{28}$$

for $k,l \in \{A,B\}$, $l \neq k$. Comparing (28) with (13), we see that: *i)* the firms' FOCs have been added to the system, making explicit—and replacing—the generic optimal quantities $\hat{y}_k[P_k, w_k]$ and $\hat{x}_k[P_l, w_k]$; and *ii)* using (3), prices p_k, s_k have been replaced by $P_k v_k'[y_k/L_k]$ and $P_l v_l'[0]$, respectively.

Next, we eliminate P_k, P_l, and n_k in (28) through a series of substitutions. Substituting FOC_y^k and FOC_x^k into ZP^k yields

$$ZP^k: \quad \frac{w_k/\phi_k}{1-\varepsilon_{v_k'}[y_k/L_k]}y_k + \frac{w_k/\phi_k}{1-\varepsilon_{v_l'}[x_k/L_l]}x_k = w_k\left\{1/\Phi_k + (y_k + x_k)/\phi_k\right\} \quad .$$

Rewriting yields the form of ZP^k in the lemma.

Similarly, substituting FOC_x^k and LM^k into BP yields

$$BP: \quad \frac{L_k}{C_k[y_k+x_k]}\frac{w_k/\phi_k}{1-\varepsilon_{v_l'}[x_k/L_l]}x_k = \frac{L_l}{C_l[y_l+x_l]}\frac{w_l/\phi_l}{1-\varepsilon_{v_k'}[x_l/L_k]}x_l \quad ,$$

which we then solve for $\frac{w_k/\phi_k}{w_l/\phi_l}$ to get the form of BP in (27).

Finally, dividing FOC_y^k by FOC_x^l yields FOC^k. ■

A.1.2 Equilibrium Quantities and U_A as $\phi_B \to 0$

Consider the equilibrium system in (27). In the following sequence of lemmas, we calculate the equilibrium values for y_k, x_k, n_k as $\phi_B \to 0$, $k \in \{A, B\}$. We also show that $\lim_{\phi_B \to 0} U_A = \dot{U}_A$.

Lemma 3

$$\lim_{\phi_B \to 0} y_B = \lim_{\phi_B \to 0} x_B = 0 \ .$$

Proof Consider ZP^B in (27). Taking the limit as $\phi_B \to 0$ yields

$$\lim_{\phi_B \to 0} m_B[y_B]\,y_B + m_A[x_B]\,x_B = 0 \ .$$

Since $m_k[z] > 0$ for $z > 0$, y_B and x_B must both go to zero as $\phi_B \to 0$. ■

Lemma 4

$$\lim_{\phi_B \to 0} y_A = \dot{y}_A \ \textit{and} \ \lim_{\phi_B \to 0} x_A = 0 \ .$$

Proof In (27), equating FOC^A with BP and isolating x_A yields

$$x_A \leq \frac{L_B}{L_A}\frac{v_A'[x_B/L_A]}{v_A'[y_A/L_A]}\frac{1-\varepsilon_{v_B'}[x_A/L_B]}{1-\varepsilon_{v_A'}[y_A/L_A]}\frac{C_A[y_A+x_A]}{C_B[y_B+x_B]}x_B \ . \tag{29}$$

Taking the limit as $\phi_B \to 0$,

$$\lim_{\phi_B \to 0} x_A \leq \frac{L_B}{L_A} \times \lim_{\phi_B \to 0}\left\{\frac{1-\varepsilon_{v_B'}[x_A/L_B]}{1-\varepsilon_{v_A'}[y_A/L_A]}\frac{C_A[y_A+x_A]}{v_A'[y_A/L_A]}\right\} \times \lim_{\phi_B \to 0}\frac{v_A'[x_B/L_A]\,x_B}{\frac{1}{\Phi_B}+\frac{y_B+x_B}{\phi_B}} = 0 \ .$$

Here we have used that the braced factor is finite, since ZP^A in (27) guarantees $0 \leq y_A, x_A < \infty$, while $\lim_{\phi_B \to 0} x_B = 0$ by Lemma 3.

Finally, using $\lim_{\phi_B \to 0} x_A = 0$, ZP^A implies that $\lim_{\phi_B \to 0} y_A = \dot{y}_A$. ■

Lemma 5

$$\lim_{\phi_B \to 0} n_A = \dot{n}_A \text{ and } \lim_{\phi_B \to 0} n_B \leq \Phi_B L_B \ .$$

Proof Using LM^k in (27), as well as $\lim_{\phi_B \to 0} x_A = \lim_{\phi_B \to 0} x_B = 0$, we find that

$$\begin{aligned}
\lim_{\phi_B \to 0} n_A &= \lim_{\phi_B \to 0} \frac{L_A}{C_A [y_A + x_A]} = \lim_{\phi_B \to 0} \frac{L_A}{\frac{1}{\Phi_A} + \dot{y}_A / \phi_A} = \dot{n}_A, \text{ and} \\
\lim_{\phi_B \to 0} n_B &= \lim_{\phi_B \to 0} \frac{L_B}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} \leq \Phi_B L_B \ .
\end{aligned}$$

■

Lemma 6

$$\lim_{\phi_B \to 0} U_A = \dot{U}_A \ .$$

Proof Together, Lemmas 3, 4, and 5 imply that

$$\begin{aligned}
\lim_{\phi_B \to 0} U_A [\phi_B] &= \lim_{\phi_B \to 0} n_A v_A [y_A / L_A] + n_B v_A [x_B / L_A] \\
&= \dot{n}_A v_A [\dot{y}_A / L_A] + v_A [0] \lim_{\phi_B \to 0} n_B = \dot{n}_A v_A [\dot{y}_A / L_A] = \dot{U}_A \ .
\end{aligned}$$

■

Lemma 7 *For $\phi_B > 0$ sufficiently small, $y_B = 0$.*

Proof If the inequality in FOC^k in (27) is strict, then y_k must be cornered at 0. Combining FOC^A and FOC^B, it therefore suffices to show that, for $\phi_B > 0$ sufficiently small,

$$\frac{R_A' [y_A]}{R_A' [x_B]} < \frac{R_B' [x_A]}{R_B' [y_B]} \ . \tag{30}$$

For the RHS of the inequality in (30), observe that

$$\lim_{\phi_B \to 0} \frac{R_B' [x_A]}{R_B' [y_B]} = \frac{R_B' [0]}{R_B' [0]} = 1 \ ,$$

while for the LHS,

$$\lim_{\phi_B \to 0} \frac{R'_A[y_A]}{R'_A[x_B]} = \frac{R'_A[\dot{y}_A]}{R'_A[0]} < 1 \ ,$$

since $R'_k[\cdot]$ is strictly decreasing. Hence, by continuity, the strict inequality in (30) must hold for $\phi_B > 0$ sufficiently small. ■

Lemma 8

$$\lim_{\phi_B \to 0} \frac{x_B}{\phi_B} = \infty \ .$$

Proof From Lemma 7 we know that, for $\phi_B > 0$ sufficiently small, $y_B = 0$. In that case, ZP^B in (27) reduces to

$$m_A[x_B]\, x_B = \phi_B / \Phi_B \ . \tag{31}$$

Rewriting,

$$\frac{x_B}{\phi_B} = \frac{1}{\Phi_B m_A[x_B]} \ .$$

From Lemma 3 we know that $\lim_{\phi_B \to 0} x_B = 0$, while $m_A[0] = 0$. Hence,

$$\lim_{\phi_B \to 0} \frac{x_B}{\phi_B} = \infty \ .$$

■

Lemma 9

$$\lim_{\phi_B \to 0} \frac{x_A}{\phi_B} = \frac{L_B}{L_A} \frac{v'_A[0]\, C_A[\dot{y}_A]}{v'_A[\dot{y}_A/L_A]} (1 + m_A[\dot{y}_A]) < \infty \ . \tag{32}$$

Proof For ϕ_B sufficiently small, $y_B = 0$ (Lemma 7). In that case,

$$C_B[y_B + x_B] = C_B[x_B] = 1/\Phi_B + x_B/\phi_B \ . \tag{33}$$

Furthermore, $y_A > 0$ (Lemma 4). Hence, (29) holds with equality. Substituting (33) into (29) and dividing by ϕ_B yields

$$\frac{x_A}{\phi_B} = \frac{L_B}{L_A} \frac{v'_A[x_B/L_A]}{v'_A[y_A/L_A]} \frac{1 + m_A[y_A]}{1 + m_B[x_A]} \frac{C_A[y_A + x_A]}{\frac{1/\Phi_B}{x_B/\phi_B} + 1} \ .$$

Taking the limit as $\phi_B \to 0$ and using Lemmas 3, 4, and 8, as well as the fact that $m_k[0] = 0$, yields

$$\lim_{\phi_B \to 0} \frac{x_A}{\phi_B} = \frac{L_B}{L_A} \frac{v'_A[0]\, C_A[\dot{y}_A]}{v'_A[\dot{y}_A/L_A]} (1 + m_A[\dot{y}_A]) < \infty \ .$$

■

A.1.3 The IFT applies for all $\phi_B > 0$ sufficiently small

From Lemmas 7 and 4, we know that $y_B = 0$ for $\phi_B > 0$ sufficiently small, while $y_A > 0$. In that case, FOC_y^B is slack while FOC_y^A is binding. The system in (27) then reduces to

$$
\begin{aligned}
&FOC^A : \frac{R'_A[y_A]}{R'_A[x_B]} = \frac{w_A/\phi_A}{w_B/\phi_B} \\
&ZP^A : m_A[y_A]\,y_A + m_B[x_A]\,x_A = \phi_A/\Phi_A \\
&ZP^B : m_A[x_B]\,x_B = \phi_B/\Phi_B \\
&BP : \frac{L_B}{L_A}\frac{C_A[y_A + x_A]}{C_B[x_B]}\frac{1 + m_A[x_B]}{1 + m_B[x_A]}\frac{x_B}{x_A} = \frac{w_A/\phi_A}{w_B/\phi_B} \; .
\end{aligned}
$$

Equating the LHS of FOC^A with that of BP further reduces the system to one that is solely a function of y_A, x_A, and x_B :

$$
\begin{aligned}
&ZP^A : m_A[y_A]\,y_A + m_B[x_A]\,x_A - \phi_A/\Phi_A = 0 \\
&ZP^B : m_A[x_B]\,x_B - \phi_B/\Phi_B = 0 \\
&BP : K[x_B]\frac{C_A[y_A + x_A]}{(1 + m_B[x_A])\,x_A} - \frac{R'_A[y_A]}{R'_A[x_B]} = 0 \; ,
\end{aligned}
\tag{34}
$$

where

$$K[x_B] \equiv \frac{L_B}{L_A}\frac{1 + m_A[x_B]}{C_B[x_B]}x_B > 0$$

It can be easily verified that ZP^A, ZP^B, BP in (34) are continuously differentiable in y_A, x_A, and x_B, while we know from Proposition 1 that the system always has a solution. In order to apply the IFT (see, e.g., Rudin, 1976), it remains to verify that the Jacobian is non-singular.

Since $\frac{\partial ZP^B}{\partial y_A} = \frac{\partial ZP^A}{\partial x_B} = \frac{\partial ZP^B}{\partial x_A} = 0$, the Jacobian of (34) is

$$
J = \begin{bmatrix}
\frac{\partial ZP^A}{\partial y_A} & \frac{\partial ZP^A}{\partial x_A} & 0 \\
0 & 0 & \frac{\partial ZP^B}{\partial x_B} \\
\frac{\partial BP}{\partial y_A} & \frac{\partial BP}{\partial x_A} & \frac{\partial BP}{\partial x_B}
\end{bmatrix} .
$$

Lemma 10

$$\det J \neq 0 \; .$$

Proof Straight forward calculation shows that

$$\det J = -\frac{\partial ZP^B}{\partial x_B}\left(\frac{\partial ZP^A}{\partial y_A}\frac{\partial BP}{\partial x_A} - \frac{\partial ZP^A}{\partial x_A}\frac{\partial BP}{\partial y_A}\right) .$$

Observe that

$$\frac{\partial ZP^B}{\partial x_B} = m_A[x_B] + m'_A[x_B]\,x_B = \frac{\varepsilon_{v'_A}(x_B/L_A)}{1-\varepsilon_{v'_A}(x_B/L_A)} + \frac{x_B}{L_A}\frac{\varepsilon'_{v'_A}(x_B/L_A)}{\left(1-\varepsilon_{v'_A}(x_B/L_A)\right)^2} > 0 ,$$

where the inequality follows from $x_B > 0$. Similarly, $\frac{\partial ZP^A}{\partial y_A} > 0$ and $\frac{\partial ZP^A}{\partial x_A} > 0$. Furthermore,

$$\frac{\partial BP}{\partial y_A} = \frac{K[x_B]}{\phi_A(1+m_B[x_A])\,x_A} - \frac{R''_A[y_A]}{R'_A[x_B]} > 0 ,$$

where the inequality follows from $K[x_B], R'_A[x_B] > 0$ and $R''_A[y_A] < 0$. Finally,

$$\begin{aligned}\frac{\partial BP}{\partial x_A} &= K[x_B]\frac{\partial}{\partial x_A}\left[\frac{C_A[y_A+x_A]}{(1+m_B[x_A])\,x_A}\right] \\ &= K[x_B]\frac{(1+m_B[x_A])(x_A/\phi_A) - C_A[y_A+x_A](1+m_B[x_A]+m'_B[x_A]\,x_A)}{((1+m_B[x_A])\,x_A)^2} ,\end{aligned}$$

which takes the sign of

$$(1+m_B)(-F_A - y_A/\phi_A) - C_A[y_A+x_A]\,m'_B x_A < 0 ,$$

Collecting signs,

$$\frac{\partial ZP^B}{\partial x_B}, \frac{\partial ZP^A}{\partial y_A}, \frac{\partial ZP^A}{\partial x_A}, \frac{\partial BP}{\partial y_A} > 0, \text{ and } \frac{\partial BP}{\partial x_A} < 0 .$$

Therefore,

$$\det J = -\frac{\partial ZP^B}{\partial x_B}\left(\frac{\partial ZP^A}{\partial y_A}\frac{\partial BP}{\partial x_A} - \frac{\partial ZP^A}{\partial x_A}\frac{\partial BP}{\partial y_A}\right) < 0 .$$

This proves the claim. ■

The IFT allows us to implicitly differentiate the system of equilibrium equations for $\phi_B > 0$ sufficiently small, which we undertake in the next section.

A.1.4 Derivatives as $\phi_B \to 0$

For future reference, we show

Lemma 11

$$z\varepsilon'_{v'_k}[z] = \varepsilon_{v'_k}[z]\left(1 + \varepsilon_{v''_k}[z] + \varepsilon_{v'_k}[z]\right)$$

Proof Tedious but trivial. ■

In the following sequence of lemmas, we study the limits of the derivatives of y_k, x_k, n_k with respect to ϕ_B, as $\phi_B \to 0$.

Lemma 12

$$\lim_{\phi_B \to 0} \frac{dx_A}{d\phi_B} = \lim_{\phi_B \to 0} \frac{x_A}{\phi_B} < \infty \ .$$

Proof Consider x_A as a function of ϕ_B. From the definition of a derivative,

$$\left.\frac{dx_A}{d\phi_B}\right|_{\phi_B = k} = \lim_{h \to 0} \frac{x_A[k+h] - x_A[k]}{(k+h) - k} \ .$$

Taking the limit as $k \to 0$,

$$\begin{aligned}\lim_{k \to 0} \left.\frac{dx_A}{d\phi_B}\right|_{\phi_B = k} &= \lim_{k \to 0}\left(\lim_{h \to 0} \frac{x_A[k+h] - x_A[k]}{(k+h) - k}\right) = \lim_{h \to 0}\left(\lim_{k \to 0} \frac{x_A[k+h] - x_A[k]}{(k+h) - k}\right) \\ &= \lim_{h \to 0} \frac{x_A[h] - x_A[0]}{h} = \lim_{h \to 0} \frac{x_A[h]}{h} \ .\end{aligned}$$

Here we have used the Moore-Osgood Theorem to interchange limits. Finiteness now follows from Lemma 9. ■

Lemma 13

$$\lim_{\phi_B \to 0} \frac{dy_A}{d\phi_B} = 0 \ .$$

Proof Implicitly differentiating ZP^A in (27) with respect to ϕ_B, we find

$$\left(m_A[y_A] + m'_A[y_A]\, y_A\right)\frac{dy_A}{d\phi_B} + \left(m_B[x_A] + m'_B[x_A]\, x_A\right)\frac{dx_A}{d\phi_B} = 0 \ .$$

Recall that $\lim_{\phi_B \to 0} y_A = \dot{y}_A > 0$, while $\lim_{\phi_B \to 0} x_A = \dot{x}_A = 0 = m_B[0]$. Using $\lim_{\phi_B \to \infty} dx_A/d\phi_B < \infty$ (Lemma 12), it then follows that

$$\left(m_A[\dot{y}_A] + m'_A[\dot{y}_A]\, \dot{y}_A\right)\lim_{\phi_B \to \infty} \frac{dy_A}{d\phi_B} = 0 \ .$$

Therefore, $\lim_{\phi_B \to \infty} dy_A/d\phi_B = 0$. ■

Lemma 14

$$\lim_{\phi_B \to 0} n_B \frac{dx_B}{d\phi_B} = \frac{L_B}{2} \ .$$

Proof Differentiating (31) with respect to ϕ_B,

$$\left(m_B \left[x_B\right] + m'_B \left[x_B\right] x_B\right) \frac{dx_B}{d\phi_B} = 1/\Phi_B \ .$$

Isolating $dx_B/d\phi_B$ and writing out $m_B \left[x_B\right], m'_B \left[x_B\right]$,

$$\frac{dx_B}{d\phi_B} = \frac{1}{\Phi_B} \frac{1 - \varepsilon_{v'_A} \left[x_B/L_A\right]}{\frac{\varepsilon'_{v'_A}\left[x_B/L_A\right]}{1-\varepsilon_{v'_A}\left[x_B/L_A\right]} \frac{x_B}{L_A} + \varepsilon_{v'_A} \left[x_B/L_A\right]} \ . \tag{35}$$

Using that $n_B = L_B / \left(1/\Phi_B + x_B/\phi_B\right)$, which follows from LM^B in (27),

$$n_B \frac{dx_B}{d\phi_B} = \frac{L_B \frac{\phi_B}{\Phi_B}}{\frac{\phi_B}{\Phi_B} + x_B} \frac{1 - \varepsilon_{v'_A} \left[x_B/L_A\right]}{\frac{\varepsilon'_{v'_A}\left[x_B/L_A\right]}{1-\varepsilon_{v'_A}\left[x_B/L_A\right]} \frac{x_B}{L_A} + \varepsilon_{v'_A} \left[x_B/L_A\right]} \ .$$

Reusing (31) and simplifying,

$$n_B \frac{dx_B}{d\phi_B} = \frac{L_B \left(1 - \varepsilon_{v'_A} \left[x_B/L_A\right]\right)}{\frac{(x_B/L_A)\varepsilon'_{v'_A}\left[x_B/L_A\right]}{\varepsilon_{v'_A}\left[x_B/L_A\right]} \frac{1}{1-\varepsilon_{v'_A}\left[x_B/L_A\right]} + 1} \ . \tag{36}$$

Using Lemma 11, $\lim_{\phi_B \to 0} x_B = 0$, $\lim_{\phi_B \to 0} x_B/\phi_B = \infty$, and $\varepsilon_{v''_A}[0] = \varepsilon_{v'_A}[0] = 0$, we find

$$\lim_{\phi_B \to 0} n_B \frac{dx_B}{d\phi_B} = \frac{L_B}{1+1} = \frac{L_B}{2} \ .$$

■

Lemma 15

$$\lim_{\phi_B \to 0} \frac{\phi_B}{x_B} \frac{dx_B}{d\phi_B} = \frac{1}{2} \ .$$

Proof From (35),

$$\frac{\phi_B}{x_B} \frac{dx_B}{d\phi_B} = \frac{1}{x_B} \frac{\phi_B}{\Phi_B} \frac{1 - \varepsilon_{v'_A} \left[x_B/L_A\right]}{\frac{\varepsilon'_{v'_A}\left[x_B/L_A\right]}{1-\varepsilon_{v'_A}\left[x_B/L_A\right]} \frac{x_B}{L_A} + \varepsilon_{v'_A} \left[x_B/L_A\right]} \ .$$

Using (31) and simplifying,

$$\frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}=\frac{1}{\frac{(x_B/L_A)\varepsilon'_{v'_A}[x_B/L_A]}{\varepsilon_{v'_A}[x_B/L_A]}\frac{1}{1-\varepsilon_{v'_A}[x_B/L_A]}+1}\ .$$

Using Lemma 11, $\lim_{\phi_B\to 0}x_B=0$, and $\varepsilon_{v''_A}[0]=\varepsilon_{v'_A}[0]=0$, it now follows that

$$\lim_{\phi_B\to 0}\frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}=\frac{1}{1+1}=\frac{1}{2}\ .$$

■

Lemma 16

$$\lim_{\phi_B\to 0}\frac{dn_A}{d\phi_B}=-\frac{L_B}{\dot{y}_A}\frac{v'_A[0]}{v'_A[\dot{y}_A/L_A]}<0\ . \tag{37}$$

Proof Recall from LM^A in (27) that $n_A=L_A/C_A[y_A+x_A]$. Differentiating with respect to ϕ_B yields

$$\frac{dn_A}{d\phi_B}=-\frac{L_A}{C_A^2[y_A+x_A]}\frac{1}{\phi_A}\left(\frac{dy_A}{d\phi_B}+\frac{dx_A}{d\phi_B}\right)\ .$$

Recall from Lemma 13 that $\lim_{\phi_B\to 0}dy_A/d\phi_B=0$ and from Lemma 4 that $\lim_{\phi_B\to 0}x_A=0$. Furthermore, jointly, Lemmas 9 and 12 imply that

$$\lim_{\phi_B\to 0}\frac{dx_A}{d\phi_B}=\frac{L_B}{L_A}\frac{v'_A[0]\,C_A[\dot{y}_A]}{v'_A[\dot{y}_A/L_A]}\left(1+m_A[\dot{y}_A]\right)\ .$$

Therefore,

$$\lim_{\phi_B\to 0}\frac{dn_A}{d\phi_B}=-\frac{L_B}{\phi_A C_A[\dot{y}_A]}\frac{v'_A[0]}{v'_A[\dot{y}_A/L_A]}\left(1+m_A[\dot{y}_A]\right)\ .$$

Finally, using that $1/(\phi_A C_A[\dot{y}_A])=\left(1-\varepsilon_{v'_A}[\dot{y}_A/L_A]\right)/\dot{y}_A$, which follows from zero profits, we find the expression in (37). ■

Lemma 17

$$\lim_{\phi_B\to 0}\frac{dn_B}{d\phi_B}x_B=\frac{L_B}{2}\ .$$

Proof For ϕ_B sufficiently small, $y_B=0$ (Lemma 7). LM^B in (27) then reduces to,

$$n_B=\frac{\phi_B L_B}{\frac{\phi_B}{\Phi_B}+x_B}\ .$$

Differentiating with respect to ϕ_B,

$$\frac{dn_B}{d\phi_B} = \frac{L_B}{\frac{\phi_B}{\Phi_B} + x_B} - \frac{\phi_B L_B}{\frac{\phi_B}{\Phi_B} + x_B} \frac{\frac{1}{\Phi_B} + \frac{dx_B}{d\phi_B}}{\frac{\phi_B}{\Phi_B} + x_B} = x_B L_B \frac{1 - \frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}}{\left(\frac{\phi_B}{\Phi_B} + x_B\right)^2} \ .$$

Using (31),

$$\frac{dn_B}{d\phi_B} = x_B L_B \frac{1 - \frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}}{\left\{(1 + m_A[x_B])\, x_B\right\}^2} = \frac{L_B}{x_B} \frac{1 - \frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}}{(1 + m_A[x_B])^2} \ .$$

Hence,

$$\frac{dn_B}{d\phi_B} x_B = L_B \frac{1 - \frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B}}{(1 + m_A[x_B])^2} \ .$$

Finally, using $\lim_{\phi_B \to 0} \frac{\phi_B}{x_B}\frac{dx_B}{d\phi_B} = \frac{1}{2}$, $\lim_{\phi_B \to 0} x_B/\phi_B = \infty$, and $\lim_{\phi_B \to 0} x_B = 0 = m_A[0]$, from Lemmas 15, 8, 3, respectively, we find that

$$\lim_{\phi_B \to 0} \frac{dn_B}{d\phi_B} x_B = L_B \frac{1 - \frac{1}{2}}{(1)^2} = \frac{L_B}{2} \ .$$

■

A.1.5 Signing $\lim_{\phi_B \to 0} dU_A/d\phi_B$

Lemma 18

$$\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} < 0 \ .$$

Proof Utility in country A is

$$U_A = n_A v_A[y_A/L_A] + n_B v_A[x_B/L_A] \ .$$

Differentiating with respect to ϕ_B yields

$$\frac{dU_A}{d\phi_B} = \frac{dn_A}{d\phi_B} v_A[y_A/L_A] + n_A v_A'[y_A/L_A] \frac{1}{L_A}\frac{dy_A}{d\phi_B} + \frac{dn_B}{d\phi_B} v_A[x_B/L_A] + n_B v_A'[x_B/L_A] \frac{1}{L_A}\frac{dx_B}{d\phi_B} \ ,$$

which we can write as

$$\frac{dn_A}{d\phi_B} v_A[y_A/L_A] + n_A v_A'[y_A/L_A] \frac{1}{L_A}\frac{dy_A}{d\phi_B} + \frac{x_B}{L_A}\frac{dn_B}{d\phi_B}\frac{v_A[x_B/L_A]}{x_B/L_A} + n_B v_A'[x_B/L_A] \frac{1}{L_A}\frac{dx_B}{d\phi_B} \ .$$

Using $\lim_{\phi_B \to 0} x_B = \lim_{\phi_B \to 0} dy_A/d\phi_B = v_A[0] = 0$ (Lemmas 3 and 13), $\lim_{\phi_B \to 0} \frac{dn_B}{d\phi_B} x_B = \lim_{\phi_B \to 0} n_B \frac{dx_B}{d\phi_B} = \frac{L_B}{2}$ (Lemmas 17 and 14), $\lim_{z \to 0} v[z]/z = v'[0]$, and the expression for $\lim_{\phi_B \to 0} \frac{dn_A}{d\phi_B}$

in (37) yields

$$\lim_{\phi_B \to 0} \frac{dU_A}{d\phi_B} = -\frac{L_B}{\dot{y}_A} \frac{v'_A [0]}{v'_A [\dot{y}_A/L_A]} v_A [\dot{y}_A/L_A] + 0 + \frac{1}{2}\frac{L_B}{L_A} v'_A [0] + \frac{1}{2}\frac{L_B}{L_A} v'_A [0]$$

$$= -\frac{L_B}{L_A} \frac{v'_A [0]}{\varepsilon_{v_A} [\dot{y}_A/L_A]} + \frac{L_B}{L_A} v'_A [0] = \left(1 - \frac{1}{\varepsilon_{v_A} [\dot{y}_A/L_A]}\right) \frac{L_B}{L_A} v'_A [0] < 0 \ ,$$

where the inequality follows from $\varepsilon_{v_A} [\dot{y}_A/L_A] < 1$ (Lemma 1). ■

Together, Lemmas 6 and 18 imply Theorem 1 ■

A.2 Proof of Proposition 2

A.2.1 Equilibrium under CES

Lemma 19 *Under symmetric CES utility, equilibrium is given by*

$$y_A = \frac{\rho}{1-\rho} \frac{1}{1+G} \frac{\phi_A}{\Phi_A} \text{ and } x_A = \frac{\rho}{1-\rho} \frac{G}{1+G} \frac{\phi_A}{\Phi_A}$$

$$y_B = \frac{\rho}{1-\rho} \frac{G}{1+G} \frac{\phi_B}{\Phi_B} \text{ and } x_B = \frac{\rho}{1-\rho} \frac{1}{1+G} \frac{\phi_B}{\Phi_B}$$

$$n_A = (1-\rho) L_A \Phi_A \text{ and } n_B = (1-\rho) L_B \Phi_B$$

where $G \equiv \left(\frac{L_B}{L_A}\right)^{\frac{1}{\rho}} \left(\frac{\Phi_B}{\Phi_A}\right)^{\frac{1-\rho}{\rho}} \frac{\phi_B}{\phi_A}$.

Proof Infinite marginal utility at zero implies that y_A, x_A, y_B, x_B are strictly positive in equilibrium. Hence, all FOCs must hold with equality. The equilibrium system (27) then reduces to

$$FOC : \frac{x_B}{y_A} = \frac{y_B}{x_A}$$

$$ZP_k : y_A + x_A = \frac{\rho}{1-\rho} \frac{\phi_A}{\Phi_A} \text{ and } y_B + x_B = \frac{\rho}{1-\rho} \frac{\phi_B}{\Phi_B}$$

$$BP : \frac{L_B}{L_A} \frac{\phi_A/\Phi_A + y_A + x_A}{\phi_B/\Phi_B + y_B + x_B} \frac{x_B}{x_A} = \frac{\phi_A}{\phi_B} \left(\frac{x_B}{y_A}\right)^{1-\rho}$$

Solving the system is straightforward. ■

A.2.2 $\phi_B \to 0$ under CES

Lemma 19 implies

Corollary 1 *Under symmetric CES,*

$$\begin{aligned}\lim_{\phi_B\to 0} y_A &= \dot{y}_A \text{ and } \lim_{\phi_B\to 0} x_A = 0\\ \lim_{\phi_B\to 0} y_B &= 0 \text{ and } \lim_{\phi_B\to 0} x_B = 0\end{aligned}$$

Lemma 20 *Under symmetric CES,*

$$\begin{aligned}\lim_{\phi_B\to 0}\frac{dy_A}{d\phi_B} &= -\infty = \lim_{\phi_B\to 0}\frac{dP_A}{d\phi_B} \text{ and } \lim_{\phi_B\to 0}\frac{dx_A}{d\phi_B} = \infty\\ \lim_{\phi_B\to 0}\frac{dy_B}{d\phi_B} &= 0 \text{ and } \lim_{\phi_B\to 0}\frac{dx_B}{d\phi_B} = \frac{\rho}{1-\rho}\frac{1}{\Phi_B}\\ \frac{dn_A}{d\phi_B} &= \frac{dn_B}{d\phi_B} = \frac{dp_A}{d\phi_B} = 0\end{aligned}$$

Proof Trivial. ■

Lemma 21 *Under symmetric CES,*

$$\lim_{\phi_B\to 0}\frac{dU_A}{d\phi_B} = \infty\ .$$

Proof Straight forward calculations show that

$$\begin{aligned}U_A &= n_A\left(\frac{\rho}{1-\rho}\frac{1}{1+G}\frac{\phi_A}{\Phi_A}\frac{1}{L_A}\right)^{\rho} + n_B\left(\frac{\rho}{1-\rho}\frac{1}{1+G}\frac{\phi_B}{\Phi_B}\frac{1}{L_A}\right)^{\rho}\\ &= \left\{n_A\left(\frac{\phi_A}{\Phi_A}\right)^{\rho} + n_B\left(\frac{\phi_B}{\Phi_B}\right)^{\rho}\right\}\frac{1}{(1+G)^{\rho}}\left(\frac{\rho}{1-\rho}\frac{1}{L_A}\right)^{\rho}\ .\end{aligned}$$

Hence,

$$\frac{dU_A}{d\phi_B} = \left[n_B\left(\frac{\phi_B}{\Phi_B}\right)^{\rho-1}\frac{1}{\Phi_B} - \left\{n_A\left(\frac{\phi_A}{\Phi_A}\right)^{\rho} + n_B\left(\frac{\phi_B}{\Phi_B}\right)^{\rho}\right\}\frac{dG/d\phi_B}{1+G}\right]\rho\left(\frac{1}{1+G}\frac{\rho}{1-\rho}\frac{1}{L_A}\right)^{\rho}\ ,$$

where

$$\frac{dG}{d\phi_B} = \left(\frac{L_B}{L_A}\right)^{\frac{1}{\rho}}\left(\frac{\Phi_B}{\Phi_A}\right)^{\frac{1-\rho}{\rho}}\frac{1}{\phi_A}\ .$$

Taking limits,

$$\lim_{\phi_B\to 0}\frac{dU_A}{d\phi_B} = \left\{n_B\left(\frac{\Phi_B}{\lim_{\phi_B\to 0}\phi_B}\right)^{1-\rho}\frac{1}{\Phi_B} - n_A\left(\frac{\phi_A}{\Phi_A}\right)^{\rho}\frac{dG}{d\phi_B}\right\}\rho\left(\frac{\rho}{1-\rho}\frac{1}{L_A}\right)^{\rho} = \infty\ .$$

■

B Letting $\Phi_B \to 0$

Define

$$\varepsilon_{v'_k}^{-1}[1] \equiv \sup\left\{z \in [0,\infty] \,|\varepsilon_{v'_k}[z] < 1\right\} \in (0,\infty] \ ,$$

and observe that

$$\varepsilon_{v'_k}^{-1}[1] > \dot{y}_k \ .$$

With slight abuse of notation, let $\dot{x}_A$ denote the unique solution to

$$m_B [x_A] \, x_A = \phi_A / \Phi_A \ .$$

Observe that $\dot{x}_A$ exists, is independent of Φ_B and bounded.

In the following sequence of lemmas, we calculate the equilibrium values for y_k, x_k, n_k as $\Phi_B \to 0$, $k \in \{A, B\}$, and show that $\lim_{\phi_B \to 0} U_A = \dot{U}_A$.

Lemma 22 *$y_A \in [0, \dot{y}_A]$ and $x_A \in [0, \dot{x}_A]$. Hence, y_A and x_A remain bounded as $\Phi_B \to 0$.*

Proof The claims follow immediately from $\lim_{z\to\infty} m_k [z] \, z = \infty$ and

$$ZP_A : m_A [y_A] \, y_A + m_B [x_A] \, x_A = \phi_A / \Phi_A \ .$$

■

Lemma 23 $\lim_{\Phi_B \to 0} y_A = \dot{y}_A$ *and* $\lim_{\Phi_B \to 0} x_A = 0$.

Proof In (27), equating FOC^A with BP and isolating x_A yields

$$x_A \leq \frac{L_B}{L_A} \frac{v'_A [x_B/L_A]}{v'_A [y_A/L_A]} \frac{1 - \varepsilon_{v'_B} [x_A/L_B]}{1 - \varepsilon_{v'_A} [y_A/L_A]} \frac{C_A [y_A + x_A]}{C_B [y_B + x_B]} x_B \ .$$

Taking the limit as $\Phi_B \to 0$,

$$\lim_{\Phi_B \to 0} x_A \leq \frac{L_B}{L_A} \times \lim_{\Phi_B \to 0} \left\{ \frac{1 - \varepsilon_{v'_B} [x_A/L_B]}{1 - \varepsilon_{v'_A} [y_A/L_A]} \frac{C_A [y_A + x_A]}{v'_A [y_A/L_A]} \right\} \times \lim_{\Phi_B \to 0} \frac{v'_A [x_B/L_A] \, x_B}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} \ . \tag{38}$$

The braced factor is finite, since Lemma 22 guarantees that $0 \leq y_A \leq \dot{y}_A$ and $0 \leq x_A < \dot{x}_A$. If $\lim_{\Phi_B \to 0} x_B < \infty$, then the last factor goes to zero as $\Phi_B \to 0$. Hence, $\lim_{\Phi_B \to 0} x_A = 0$. If

$\lim_{\Phi_B \to 0} x_B = \infty$, then write (38) as

$$\begin{aligned}\lim_{\Phi_B \to 0} x_A &\leq \frac{L_B}{L_A} \times \lim_{\Phi_B \to 0} \left\{ \frac{1 - \varepsilon_{v'_B}[x_A/L_B]}{1 - \varepsilon_{v'_A}[y_A/L_A]} \frac{C_A[y_A + x_A]}{v'_A[y_A/L_A]} \right\} \times \lim_{\Phi_B \to 0} \frac{v'_A[x_B/L_A]}{\frac{1}{x_B \Phi_B} + \frac{\frac{y_B}{x_B}+1}{\phi_B}} \\ &\leq \frac{L_B}{L_A} \times \lim_{\Phi_B \to 0} \left\{ \frac{1 - \varepsilon_{v'_B}[x_A/L_B]}{1 - \varepsilon_{v'_A}[y_A/L_A]} \frac{C_A[y_A + x_A]}{v'_A[y_A/L_A]} \right\} \times \lim_{\Phi_B \to 0} \frac{v'_A[x_B/L_A]}{\frac{1}{\phi_B}} = 0 \ ,\end{aligned}$$

where we have used that 1) $\frac{1}{x_B \Phi_B} + \frac{\frac{y_B}{x_B}+1}{\phi_B} > \frac{1}{\phi_B}$; 2) $\lim_{\Phi_B \to 0} x_B = \infty$, and 3) $\lim_{z \to \infty} v'_k[z] = 0$.

Finally, using $\lim_{\Phi_B \to 0} x_A = 0$, ZP^A implies that $\lim_{\Phi_B \to 0} y_A = \dot{y}_A$. ■

Lemma 24 *1)* $\lim_{\Phi_B \to 0} n_A = \dot{n}_A$*; 2)* $\lim_{\Phi_B \to 0} n_B = 0$*; and 3)* $\lim_{\Phi_B \to 0} n_B v_A[x_B/L_A] = 0$

Proof Observe

$$LM^A : n_A C_A[y_A + x_A] = L_A \iff n_A = \frac{L_A}{C_A[y_A + x_A]} \ .$$

Using Lemma 23,

$$\lim_{\Phi_B \to 0} n_A = \lim_{\Phi_B \to 0} \frac{L_A}{C_A[y_A + x_A]} = \frac{L_A}{C_A[\dot{y}_A]} = \dot{n}_A \ .$$

Next,

$$LM^B : n_B C_B[y_B + x_B] = L_B \iff n_B = \frac{L_B}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} \ .$$

Hence,

$$\lim_{\Phi_B \to 0} n_B = \lim_{\Phi_B \to 0} \frac{L_B}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} \leq \lim_{\Phi_B \to 0} \frac{L_B}{\frac{1}{\Phi_B}} = 0 \ .$$

Finally, since $n_B = \frac{L_B}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}}$,

$$n_B v_A[x_B/L_A] = \frac{L_B v_A[x_B/L_A]}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} \ .$$

If $\lim_{\Phi_B \to 0} x_B < \infty$, then the expression goes to zero as $\Phi_B \to 0$. If $\lim_{\Phi_B \to 0} x_B = \infty$, then we write

$$\lim_{\Phi_B \to 0} n_B v_A[x_B/L_A] = \lim_{\Phi_B \to 0} \frac{L_B v_A[x_B/L_A]}{\frac{1}{\Phi_B} + \frac{y_B + x_B}{\phi_B}} = \lim_{\Phi_B \to 0} \frac{\phi_B L_B v_A[x_B/L_A]}{\frac{\phi_B}{\Phi_B} + y_B + x_B} < \phi_B L_B \lim_{\Phi_B \to 0} \frac{v_A[x_B/L_A]}{x_B} \ .$$

Using Hopital's rule, this is equal to

$$\phi_B \frac{L_B}{L_A} \lim_{\Phi_B \to 0} \frac{v'_A[x_B/L_A] \frac{dx_B}{d\Phi_B}}{\frac{dx_B}{d\Phi_B}} = \phi_B \frac{L_B}{L_A} \lim_{\Phi_B \to 0} v'_A[x_B/L_A] = 0 \ .$$

■

Lemma 25 $\lim_{\Phi_B \to 0} U_A = \dot{U}_A$.

Proof Together, Lemmas 3, 4, and 5 imply that

$$\lim_{\Phi_B \to 0} U_A [\phi_B] = \lim_{\Phi_B \to 0} n_A v_A [y_A/L_A] + \lim_{\Phi_B \to 0} n_B v_A [x_B/L_A] = \dot{n}_A v_A [\dot{y}_A/L_A] = \dot{U}_A \ .$$

■

Lemma 26 $\lim_{\Phi_B \to 0} y_B = \varepsilon^{-1}_{v'_B} [1]$ *or* $\lim_{\Phi_B \to 0} x_B = \varepsilon^{-1}_{v'_A} [1]$.

Proof The claim follows immediately from

$$ZP_B : m_B [y_B] \, y_B + m_A [x_B] \, x_B = \phi_B / \Phi_B \ ,$$

where the RHS $\to \infty$ as $\Phi_B \to 0$. ■

Lemma 27 $\lim_{\Phi_B \to 0} x_B = \varepsilon^{-1}_{v'_A} [1]$.

Proof If $\lim_{\Phi_B \to 0} y_B \neq \varepsilon^{-1}_{v'_B} [1]$, then the result follows immediately from Lemma 26. If $\lim_{\Phi_B \to 0} y_B = \varepsilon^{-1}_{v'_B} [1] > 0$, then $y_B > 0$ for small Φ_B. In that case, FOC^B must hold with equality. Since $\lim_{\Phi_B \to 0} y_A = \dot{y}_A > 0$, we have $y_A > 0$ for Φ_B sufficiently small, meaning that FOC^A also holds with equality. Equating $FOC^A$with FOC^B then yields

$$\frac{R'_A [y_A]}{R'_A [x_B]} = \frac{R'_B [x_A]}{R'_B [y_B]} \iff \frac{R'_A [x_B]}{R'_B [y_B]} = \frac{R'_A [y_A]}{R'_B [x_A]} \ . \tag{39}$$

Now suppose, by contradiction, that $\lim_{\Phi_B \to 0} x_B \neq \varepsilon^{-1}_{v'_B} [1]$. Then

$$\lim_{\Phi_B \to 0} \frac{R'_A [x_B]}{R'_B [y_B]} = \lim_{\Phi_B \to 0} \frac{v'_A \left[\frac{x_B}{L_A}\right] \left(1 - \varepsilon_{v'_A} \left[\frac{x_B}{L_A}\right]\right)}{v'_B \left[\frac{y_B}{L_B}\right] \left(1 - \varepsilon_{v'_A} \left[\frac{y_B}{L_B}\right]\right)} = \infty > \frac{R'_A [\dot{y}_A]}{R'_B [0]} = \lim_{\Phi_B \to 0} \frac{R'_A [y_A]}{R'_B [x_A]} \ ,$$

which contradicts (39). Hence, also in this case, $\lim_{\Phi_B \to 0} x_B = \varepsilon^{-1}_{v'_A} [1]$. ■

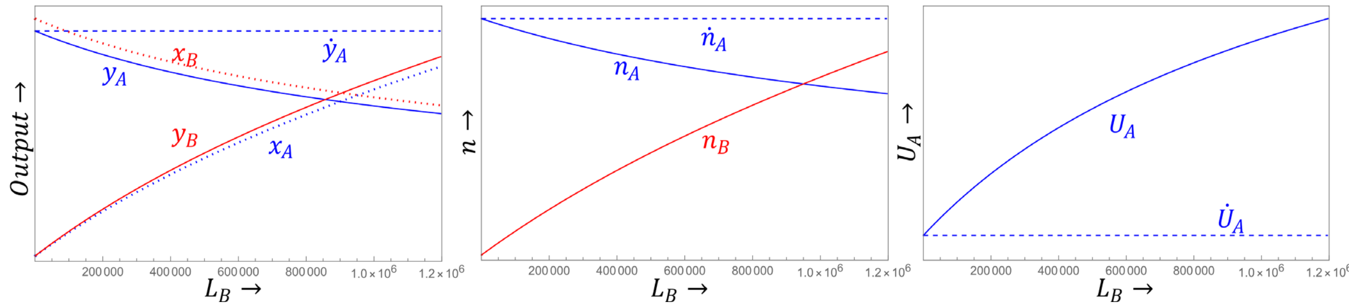

Figure 5: The figure depicts firm-level outputs (left panel), the number of firms (middle panel) and utility (right panel) as a function of population size L_B. Preferences are as in the main example in Section 3.1. .

C Example: Letting $L_B \to 0$

Figure 5 depicts per-firm output levels, the number of firms, and utility as a function of L_B. Productivity in A is slightly different from B ($\phi_A = 1 < \phi_B = 10/9$), so that differences between y_k and x_l can be perceived. Other model parameters and preferences are as in the main example of Section 3.1.

Figure 5 suggests that the effects from L_B rising above zero are similar to those from an increase in Φ_B. The drop in P_A (not shown) associated with the fall in y_A is welfare enhancing, while the decline in x_B is inconsequential, since no spending occurred. Here, $\lim_{L_B \to 0} x_B > \dot{y}_A$, because $\phi_B/\Phi_B > \phi_A/\Phi_A$. Hence, the welfare effect of exchanging domestic varieties for foreign varieties is negative. Still, free and costless trade trumps autarky for small L_B. Furthermore, despite the potential ambiguity, we have yet to find an example where country A loses from trade.