

Supplement to “Informed intermediaries”

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APPENDIX: PROOF OF PROPOSITION 2 (CONTINUED)

S.1 *Guesses*

Surplus sharing:

$$\begin{aligned}\beta(I, v_s, U, v_b) &= \beta^I \geq \frac{1}{2}, & \beta(U, v_s, I, v_b) &= 1 - \beta^I \leq \frac{1}{2}, \\ \beta(I, v_s, I, v_b) &= \frac{1}{2}.\end{aligned}$$

Efficient trading if and only if an informed agent is involved:

$$\mathcal{I}(i_s, v_s, i_b, v_b) = 1 \quad \Leftrightarrow \quad (i_s, i_b) \neq (U, U) \quad \text{and} \quad V_{v_s 0}^{i_s} - V_{v_s 1}^{i_s} + V_{v_b 1}^{i_b} - V_{v_b 0}^{i_b} > 0.$$

S.2 *Stationary distribution*

Given the guesses for $\mathcal{I}$, the inflow equal to outflow equations for the stationary distribution become

$$\mu_{L1}^U (\eta + \lambda(\mu_{H0}^I + \mu_{L0}^I)) = \eta \mu_{L0}^U, \quad (\text{S.1})$$

$$\mu_{H0}^U (\eta + \lambda(\mu_{L1}^I + \mu_{H1}^I)) = \eta \mu_{H1}^U, \quad (\text{S.2})$$

$$\mu_{L1}^I (\eta + \lambda(\mu_{H0}^U + \mu_{L0}^U)) = (\eta + \lambda \mu_{L1}^U) \mu_{L0}^I, \quad (\text{S.3})$$

$$\mu_{H0}^I (\eta + \lambda(\mu_{L1}^U + \mu_{L0}^U)) = (\eta + \lambda \mu_{H0}^U) \mu_{H1}^I. \quad (\text{S.4})$$

Combine (S.1) and (S.2), and using $\mu_{L1}^U + \mu_{L0}^U = \mu_{H0}^U + \mu_{H1}^U = \frac{1-\phi}{2}$ (since half of the uninformed agents have high valuation and half have low valuation), I get

$$\mu_{L1}^U (2\eta + \lambda(\mu_{H0}^I + \mu_{L0}^I)) = \mu_{H0}^U (2\eta + \lambda(\mu_{L1}^I + \mu_{H1}^I)) = \frac{\eta(1-\phi)}{2}. \quad (\text{S.5})$$

Similarly use (S.3) and (S.4) and $\mu_{L1}^I + \mu_{L0}^I = \mu_{H0}^I + \mu_{H1}^I$ to get

$$(\eta + \lambda \mu_{L1}^U) (\mu_{H0}^I + \mu_{L0}^I) = (\eta + \lambda \mu_{H0}^U) (\mu_{H1}^I + \mu_{L1}^I). \quad (\text{S.6})$$

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Use (S.5) and (S.6) to get $\mu_{L1}^U + \mu_{L1}^I + \mu_{H1}^I = \mu_{H0}^U + \mu_{H0}^I + \mu_{L0}^I$. Combining this with (S.5) yet again, conclude that $\mu_{L1}^U = \mu_{H0}^U \equiv \hat{\mu}^U$. This, along with (S.6), implies $(\mu_{H0}^I + \mu_{L0}^I) = (\mu_{L1}^I + \mu_{H1}^I)$; hence, $\mu_{H0}^I = \mu_{L1}^I \equiv \hat{\mu}^I$. Rewrite the inflow equals outflow conditions now as

$$\hat{\mu}^U (\eta + \lambda(\phi/2)) = \eta \frac{(1 - \phi - 2\hat{\mu}^U)}{2} \hat{\mu}^I, \quad (\eta + \lambda(\hat{\mu}^U + \hat{\mu}^I)) = \frac{\phi - 2\hat{\mu}^I}{2} (\eta + \lambda\hat{\mu}^U).$$

Solving these, I get

$$\hat{\mu}^U = \frac{1 - \phi}{\frac{\lambda}{4 + \frac{1}{\eta}}}, \quad \hat{\mu}^I = -\frac{\eta + \lambda}{\lambda} \hat{\mu}^U + \sqrt{\left(\frac{\eta + \lambda\hat{\mu}^U}{\lambda}\right)^2 + \frac{\phi}{2} \frac{\eta + \lambda\hat{\mu}^U}{\lambda}}.$$

S.3 Unflagged values

Again taking into account the guesses for $\mathcal{I}$ and β , unflagged values are given by the system

$$\begin{aligned} rV_{H0}^I &= \eta(V_{H1}^I - V_{H0}^I) + \lambda\hat{\mu}^I \frac{V_{H1}^I - V_{H0}^I + V_{L0}^I - V_{L1}^I}{2} + \lambda\hat{\mu}^U \beta^I (V_{H1}^I - V_{H0}^I + V_{L0}^U - V_{L1}^U), \\ rV_{H1}^I &= \delta_H + \eta(V_{H0}^I - V_{H1}^I) + \lambda\hat{\mu}^U \beta^I (V_{H0}^I - V_{H1}^I + V_{H1}^U - V_{H0}^U), \\ rV_{L1}^I &= \delta_L + \eta(V_{L0}^I - V_{L1}^I) + \lambda\hat{\mu}^I \frac{V_{L0}^I - V_{L1}^I + V_{H1}^I - V_{H0}^I}{2} \\ &\quad + \lambda\hat{\mu}^U \beta^I (V_{L0}^I - V_{L1}^I + V_{H1}^U - V_{H0}^U), \\ rV_{L0}^I &= \eta(V_{L1}^I - V_{L0}^I) + \lambda\hat{\mu}^U \beta^I (V_{L1}^I - V_{L0}^I + V_{L0}^U - V_{L1}^U), \\ rV_{H0}^U &= \eta(V_{H1}^U - V_{H0}^U) + \lambda\hat{\mu}^I (1 - \beta^I) (V_{H1}^U - V_{H0}^U + V_{L0}^I - V_{L1}^I) \\ &\quad + \lambda\hat{\mu}^I (1 - \beta^I) (V_{H1}^U - V_{H0}^U + V_{H0}^I - V_{H1}^I), \\ rV_{H1}^U &= \delta_H + \eta(V_{H0}^U - V_{H1}^U), \\ rV_{L1}^U &= \delta_L + \eta(V_{L0}^U - V_{L1}^U) + \lambda\hat{\mu}^I (1 - \beta^I) (V_{L0}^U - V_{L1}^U + V_{H1}^I - V_{H0}^I) \\ &\quad + \lambda\hat{\mu}^I (1 - \beta^I) (V_{L0}^U - V_{L1}^U + V_{L1}^I - V_{L0}^I), \\ rV_{L0}^U &= \eta(V_{L1}^U - V_{L0}^U). \end{aligned}$$

In terms of the values of holding an asset, this system becomes

$$\begin{aligned} rS_H^I &= \delta_H - 2\eta S_H^I + \lambda\hat{\mu}^U \beta^I (S_H^U - S_H^I) + \lambda\hat{\mu}^I \frac{(S_L^I - S_H^I)}{2} + \lambda\hat{\mu}^U \beta^I (S_L^U - S_H^I), \\ rS_L^I &= \delta_L - 2\eta S_L^I + \lambda\hat{\mu}^U \beta^I (S_L^U - S_L^I) + \lambda\hat{\mu}^I \frac{(S_H^I - S_L^I)}{2} + \lambda\hat{\mu}^U \beta^I (S_H^U - S_L^I), \\ rS_H^U &= \delta_H - 2\eta S_H^U + \lambda\hat{\mu}^I (1 - \beta^I) (S_L^I - S_H^U) + \lambda \frac{\phi - 2\hat{\mu}^I}{2} (1 - \beta^I) (S_H^I - S_H^U), \end{aligned}$$

$$rS_L^U = \delta_L - 2\eta S_L^U + \lambda \hat{\mu}^I (1 - \beta^I) (S_H^I - S_L^U) + \lambda \frac{\phi - 2\hat{\mu}^I}{2} (1 - \beta^I) (S_L^I - S_L^U).$$

Add up the first two lines and the last two lines to get

$$\begin{aligned} (r + 2\eta)(S_H^I + S_L^I) &= \delta_H + \delta_L + 2\lambda \hat{\mu}^U \beta^I (S_H^U + S_L^U) - 2\lambda \hat{\mu}^U \beta^I (S_H^I + S_L^I), \\ (r + 2\eta)(S_H^U + S_L^U) &= \delta_H + \delta_L + \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^I + S_L^I) - \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^U + S_L^U). \end{aligned}$$

These imply $(S_H^I + S_L^I) = (S_H^U + S_L^U) = \frac{\delta_H + \delta_L}{r + 2\eta}$. Now from the original system, subtract the second equation from the first and the fourth from the third to find

$$\begin{aligned} (r + 2\eta)(S_H^I - S_L^I) &= \delta_H - \delta_L - \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I)(S_H^I - S_L^I), \\ (r + 2\eta)(S_H^U - S_L^U) &= \delta_H - \delta_L - \frac{\lambda \phi}{2} (1 - \beta^I) (S_H^U - S_L^U) \\ &\quad + \left(\frac{\lambda \phi}{2} - \hat{\mu}^I \right) (1 - \beta^I) (S_H^I - S_L^I). \end{aligned}$$

Rearrange these to get the expressions

$$\begin{aligned} (S_H^I - S_L^I) &= \hat{\alpha}^I (\delta_H - \delta_L), \\ (S_H^U - S_L^U) &= \hat{\alpha}^U (\delta_H - \delta_L), \end{aligned} \tag{S.7}$$

where

$$\begin{aligned} \hat{\alpha}^I &= \frac{1}{2(r + 2\eta + \lambda(2\hat{\mu}^U \beta^I + \hat{\mu}^I))}, \\ \hat{\alpha}^U &= \hat{\alpha}^U = \left[\frac{r + 2\eta + \lambda \hat{\mu}^U + \frac{\lambda \phi}{4}}{r + 2\eta + \frac{\lambda \phi}{4}} \right] \hat{\alpha}^I, \end{aligned}$$

which finally implies

$$\begin{aligned} S_H^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) + \frac{\hat{\alpha}^i}{2} (\delta_H - \delta_L), \\ S_L^i &= \left[\frac{1}{2(r + 2\eta)} \right] (\delta_H + \delta_L) - \frac{\hat{\alpha}^i}{2} (\delta_H - \delta_L). \end{aligned}$$

S.4 Flagged values

I solve for D_{H1}^I , D_{L1}^I , and D_{H1}^U . The system defining $\{D_{va}^I\}$ is

$$\begin{aligned} rD_{H1}^I &= (\eta + \lambda \hat{\mu}^U) (D_{H0}^I - D_{H1}^I), \\ rD_{L1}^I &= (\eta + \lambda \hat{\mu}^U) (D_{L0}^I - D_{L1}^I) + \lambda \hat{\mu}^I \frac{S_H^I - S_L^I}{2}, \end{aligned}$$

$$rD_{L0}^I = (\eta + \lambda\hat{\mu}^U)(D_{L1}^I - D_{L0}^I) - \lambda\hat{\mu}^U(\beta^I S_L^U + (1 - \beta^I)S_L^I),$$

$$rD_{H0}^I = (\eta + \lambda\hat{\mu}^U)(D_{H1}^I - D_{H0}^I) + \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \lambda\hat{\mu}^U(\beta^I S_H^I + (1 - \beta^I)S_L^U).$$

Combining the first equality with the fourth and the second equality with the third yields

$$r(D_{H1}^I - D_{H0}^I) = -2(\eta + \lambda\hat{\mu}^U)(D_{H1}^I - D_{H0}^I) - \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} + \lambda\hat{\mu}^U(\beta^I S_H^I + (1 - \beta^I)S_L^U),$$

$$r(D_{L1}^I - D_{L0}^I) = -2(\eta + \lambda\hat{\mu}^U)(D_{L1}^I - D_{L0}^I) + \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} + \lambda\hat{\mu}^U(\beta^I S_L^U + (1 - \beta^I)S_L^I).$$

Solve to find

$$(D_{H1}^I - D_{H0}^I) = -\frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_H^I + (1 - \beta^I)S_L^U),$$

$$(D_{L1}^I - D_{L0}^I) = \frac{\lambda\hat{\mu}^I}{(r + 2\eta + 2\lambda\hat{\mu}^U)} \frac{S_H^I - S_L^I}{2} + \frac{\lambda\hat{\mu}^U}{(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_L^U + (1 - \beta^I)S_L^I).$$

Plug this back into the original system to get

$$D_{H1}^I = \frac{\eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \left[\lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} - \lambda\hat{\mu}^U(\beta^I S_H^I + (1 - \beta^I)S_L^U) \right], \quad (\text{S.8})$$

$$\begin{aligned} D_{L1}^I &= \frac{r + \eta + \lambda\hat{\mu}^U}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} \lambda\hat{\mu}^I \frac{S_H^I - S_L^I}{2} \\ &\quad - \frac{\lambda\hat{\mu}^U(\eta + \lambda\hat{\mu}^U)}{r(r + 2\eta + 2\lambda\hat{\mu}^U)} (\beta^I S_L^U + (1 - \beta^I)S_L^I). \end{aligned} \quad (\text{S.9})$$

The system defining $\{D_{\text{va}}^U\}$ is

$$rD_{H1}^U = \eta(D_{H0}^U - D_{H1}^U),$$

$$rD_{H0}^U = \eta(D_{H1}^U - D_{H0}^U) + \lambda\hat{\mu}^I(1 - \beta^I)(S_H^U - S_L^I) + \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_H^U - S_H^I),$$

$$rD_{L1}^U = \eta(D_{L0}^U - D_{L1}^U) + \lambda\hat{\mu}^I(1 - \beta^I)(S_H^I - S_L^U) + \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_L^I - S_L^U),$$

$$rD_{L0}^U = \eta(D_{L1}^U - D_{L0}^U).$$

Subtract the second equality from the first to get

$$\begin{aligned} r(D_{H1}^U - D_{H0}^U) &= -2\eta(D_{H1}^U - D_{H0}^U) - \lambda\hat{\mu}^I(1 - \beta^I)(S_H^U - S_L^I) \\ &\quad - \lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)(S_H^U - S_H^I) \\ \Rightarrow (D_{H1}^U - D_{H0}^U) &= -\frac{\lambda\hat{\mu}^I(1 - \beta^I)}{r + 2\eta} (S_H^U - S_L^I) - \frac{\lambda(\phi/2 - \hat{\mu}^I)(1 - \beta^I)}{r + 2\eta} (S_H^U - S_H^I). \end{aligned}$$

Finally plug back into the original system to get

$$D_{H1}^U = \frac{\eta\lambda\hat{\mu}^I(1-\beta^I)}{r(r+2\eta)}(S_H^U - S_L^I) + \frac{\eta\lambda(\phi/2 - \hat{\mu}^I)(1-\beta^I)}{r(r+2\eta)}(S_H^U - S_H^I). \quad (\text{S.10})$$

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