

Risk taking and gender in hierarchies

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In a labor market hierarchy, promotions are affected by the noisiness of information about the candidates. I study the hypothesis that males are more risk taking than females, and its implications for rates of promotion and abilities of survivors. I define promotion hierarchies with and without memory, where memory means that promotion depends on the entire history of success. In both types of hierarchies, the surviving risk takers have lower average ability whenever they have a higher survival rate. Further, even if more risk takers than non risk takers are promoted in the beginning of the hierarchy, that will be reversed over time. The risk takers will eventually have a lower survival rate, but higher ability. As a consequence of these differences, the various requirements of employment law cannot simultaneously be satisfied. Further, if promotion standards are chosen to maximize profit, the standards will reflect gender in ways that are difficult to distinguish from discriminatory intent.

KEYWORDS. Labor market hierarchy, promotion, discrimination, affirmative action, hierarchy, risk taking, gender bias.

JEL CLASSIFICATION. J7.

1. INTRODUCTION

In this paper, I study the effect of risk taking on promotion in hierarchies, where promotion at each stage depends on a signal of ability. The motivation comes from a substantial body of evidence that males are more risk taking than females, and from the continuing controversy about why males and females have different patterns of success in labor markets. Granting the premise that the genders differ in risk taking, does this have explanatory power for labor markets? The answer is mixed, partly because the theorems below can be applied to labor markets in different ways.

The theorems proved below compare promotions drawn from two populations, one of which generates accurate signals of ability and the other of which generates noisy

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signals of ability. The premise is that true abilities have the same distribution in both populations, at least initially, but that agents in one population give a noisy signal to the decision maker. This is a reduced-form hypothesis that might follow from preferences and optimizing behavior, or might reflect behavior that is hard-wired. This distinction does not matter for the theorems that I prove, although it matters for the interpretation.

I introduce two types of promotion hierarchies: those with memory and those without memory. In a hierarchy without memory, promotion at stage t depends only on the signal of ability generated at stage t . With memory, promotion can depend on the entire history of signals. Hierarchies such as sports tournaments do not have memory, since survival depends only on winning the current match. Hierarchies such as academic labor markets have memory, although promotion would typically depend more heavily on current performance than on past performance. To emphasize the distinction between hierarchies with memory and those without, I assume for the case of memory that all past signals are used symmetrically. There is no extra weight given to recent performance.

In [Section 2](#), I describe a simple model of hierarchical labor markets. In [Section 3](#), I show how promotion standards interact with risk taking, and prove an impossibility result related to employment law. Even with one period of promotion, it is impossible to find a promotion policy that simultaneously (a) promotes according to gender-blind standards, (b) promotes at equal rates from the two populations, and (c) ensures that the promoted pools have equal expected abilities. The various legal principles about discrimination are in conflict.

In [Section 4](#), I study hierarchies with and without memory, focussing on two important statistical properties—survival rates and the expected abilities of survivors. With gender-blind promotion standards, males may be advantaged at the beginning due to their risk-taking behavior, but this will be reversed at the end. Further, any advantage in the promotion rate coincides with a deficit in ability. If males have a higher promotion rate at the beginning, they have lower average ability than the promoted females, but this will also be reversed at the end.

In [Section 5](#), I turn to private objectives rather than social objectives. Different standards will be used in the two cases in which (a) promotion means changing employers and (b) all promotions take place within a firm. If a firm keeps a worker for only a single period, then the promotion standards that materialize equalize the marginal abilities of promoted workers in the two populations. But if promotions are within a firm, the firm has an incentive to preserve options on the employees with noisy signals, since more information will be collected later. At early stages, the firm retains risk takers who have lower expected ability than is required of the non risk takers. If the males are the risk takers, males are favored.

Since the paper is motivated by the considerable evidence that males are more risk taking than females,¹ I refer to risk takers as males, and non risk takers as females.

¹See [Eckel and Grossman \(2002\)](#) on gambling behavior, [Bajtelsmit and Bernasek \(1996\)](#), [Jianakoplos and Bernasek \(1998\)](#), and [Charness and Gneezy \(2007\)](#) on investment behavior, [Slovic \(1966\)](#), [Mittal and Dhade \(2007\)](#), and [Ginsburg and Miller \(1982\)](#) for experiments with children, and [Espinosa and Gardeazabal \(2006\)](#)

However, although the males' risk taking may lead to noise in formal labor-market signals like exams, other social phenomena may cause females' signals to be noisier than those of males. Promotions often rely on informal evidence like letters of reference. If males are observed more closely than females, then females may have noisier signals. Informal evidence is often under the control of the observer rather than the candidate. An academic in hope of promotion cannot commandeer attendance at a seminar, and an employee cannot demonstrate competence without being given a task. If opportunities to give informal evidence are more available to males than to females, then the signals generated by males may be more accurate than those of females, and we must reverse our understanding of the theorems in this paper.

2. RISK TAKERS, NON RISK TAKERS, AND HIERARCHIES

Suppose that agents' abilities are given by a random variable A with distribution G and density g , finite variance, and support equal to the real line. Males and females have the same distribution of abilities. A male with ability $A = a$ generates a sequence of independent random signals $Z_1, Z_2, \dots, Z_t, \dots$ each with distribution $\Phi(\cdot | a)$, density $\phi(\cdot | a)$, mean a , finite variance, and support equal to the real line.

I assume throughout that the *monotone likelihood ratio property* holds:

$$\frac{\phi(z_1 | a_1)}{\phi(z_1 | a_2)} > \frac{\phi(z_2 | a_1)}{\phi(z_2 | a_2)} \text{ when } z_1 > z_2 \text{ and } a_1 > a_2.$$

I sometimes use the following more specific assumption that the signal Z is ability plus noise.

ASSUMPTION A. Conditional on $A = a$, for each t , the random variable Z_t is equal to $a + U_t$, where $U_1, U_2, \dots, U_t, \dots$ are distributed independently with bounded density ϕ^u , and $\phi(z | a) = \phi^u(z - a)$. The densities g and ϕ^u are single peaked and symmetric around 0, with finite variance and supports equal to the real line.

Promotion standards are given by a sequence of real numbers, $c = c_1, c_2, \dots, c_t, \dots$ interpreted as thresholds for promotion. Due to the monotone likelihood ratio, the threshold policy selects agents who have higher ability in expectation.

Say that the promotion standards are *gender blind* if all agents face the same promotion standards. When I do not assume gender-blind promotion standards, I refer to the males' promotion standards by $m = m_1, m_2, \dots, m_t, \dots$ and to the females' promotion standards by $f = f_1, f_2, \dots, f_t, \dots$.

Say that the promotion standards c are *bounded* if there exist $\underline{c}, \bar{c}$ such that $\underline{c} \leq c_t \leq \bar{c}$ for all t .

on gender differences in exam strategies. See [Eckel and Grossman \(2008\)](#) for a comparison of the results from field studies versus contextual environmental experiments. See [Dekel and Scotchmer \(1999\)](#) for an evolutionary argument for the gender difference.

A female with ability a *survives* to stage t if $a \geq c_d$ for each $d = 1, \dots, t$. The probability of survival $S_t^F(a, c)$ is directly an indicator function:

$$S_t^F(a, c) = \begin{cases} 0 & \text{if } a < c_d \text{ for some } d \leq t \\ 1 & \text{if } a \geq c_d \text{ for each } d \leq t. \end{cases}$$

The *survival rate of females* at stage t is

$$E[S_t^F(A, c)] = \int_{-\infty}^{\infty} S_t^F(a, c) g(a) da = 1 - G(\max_{d \leq t} \{c_d\}).$$

The expected ability of a random female survivor is

$$E^F[A | c, t] = \frac{1}{E[S_t^F(A, c)]} \int_{\max_{d \leq t} \{c_d\}}^{\infty} a g(a) da.$$

To define the survival rate of males, I distinguish between promotion hierarchies that have memory and those that do not. In a hierarchy without memory, promotion at stage t , conditional on survival to $t - 1$, depends only on the performance in period t . In a hierarchy with memory, promotion at stage t , conditional on survival to $t - 1$, depends on the cumulative performance to t . I study the special case in which promotion depends symmetrically on all the previous signals, through the mean.

2.1 Survival in hierarchies without memory

Say that a male *survives* to t in a hierarchy without memory if $Z_d \geq c_d$ for $d = 1, \dots, t$. For a male with ability $A = a$, denote the probability of survival by $S_t^M(a, c)$:

$$S_t^M(a, c) = \prod_{d=1}^t (1 - \Phi(c_d | a)).$$

The *survival rate of males* at stage t (without memory) is

$$E[S_t^M(A, c)] = \int_{-\infty}^{\infty} S_t^M(a, c) g(a) da = \int_{-\infty}^{\infty} g(a) \prod_{d=1}^t [1 - \Phi(c_d | a)] da. \quad (1)$$

The expected ability of a random male survivor at stage t is

$$E^M[A | c, t] = \frac{E[AS_t^M(A, c)]}{E[S_t^M(A, c)]} = \int_{-\infty}^{\infty} a \frac{g(a) \prod_{d=1}^t [1 - \Phi(c_d | a)]}{\int_{-\infty}^{\infty} g(a) \prod_{d=1}^t [1 - \Phi(c_d | a)] da} da. \quad (2)$$

A *marginal* survivor at stage t is an agent who generates the signal $Z_t = c_t$. The expected ability of a marginal male survivor is

$$e^M[A | c, t] = \int_{-\infty}^{\infty} a \frac{g(a) \phi(c_t | a) S_{t-1}^M(a, c)}{\int_{-\infty}^{\infty} g(a) \phi(c_t | a) S_{t-1}^M(a, c) da} da. \quad (3)$$

2.2 Survival in hierarchies with memory

For the random sequence $Z_1, Z_2, \dots, Z_t, \dots$ define the sequence of sample means $\bar{Z}^1, \bar{Z}^2, \dots, \bar{Z}^t, \dots$ where $\bar{Z}^t = (1/t) \sum_{k=1}^t Z_k$ for each t . Say that a male *survives* to t in a hierarchy with memory if $\bar{Z}^d \geq c_d$, for $d = 1, \dots, t$.

When $A = a$, denote the probability of survival to t by $\bar{\mathcal{S}}_t^M(a, c)$, which can be written as

$$\bar{\mathcal{S}}_t^M(a, c) = \int_{c_1}^{\infty} \phi(z_1 | a) \int_{2c_2 - z_1}^{\infty} \phi(z_2 | a) \cdots \int_{tc_t - \sum_{k=1}^{t-1} z_k}^{\infty} \phi(z_t | a) dz_t \dots dz_2 dz_1.$$

The *survival rate of males* at stage t (with memory) is $E[\bar{\mathcal{S}}_t^M(A, c)]$. The expected ability of a random male survivor at stage t is

$$\bar{E}^M[A | c, t] = \frac{E[A \bar{\mathcal{S}}_t^M(A, c)]}{E[\bar{\mathcal{S}}_t^M(A, c)]} = \int_{-\infty}^{\infty} a \frac{\bar{\mathcal{S}}_t^M(a, c) g(a)}{\int_{-\infty}^{\infty} \bar{\mathcal{S}}_t^M(a, c) g(a) da} da.$$

A *marginal* survivor at stage t is an agent who generates the signal $\bar{Z}^t = c_t$. With memory, the probability of being marginal at stage t depends not only on the stage- t signal Z_t , but on the accumulated success up to that point, $\sum_{k=1}^{t-1} Z_k$. Let $\bar{g}(\cdot | c, t)$ be the probability density describing the distribution of ability, conditional on surviving to $t - 1$ and being marginal at t . Then

$$\bar{g}(a | c, t) = \frac{g(a) \int_{c_1}^{\infty} \phi(z_1 | a) \cdots \int_{(t-1)c_t - \sum_{k=1}^{t-2} z_k}^{\infty} \phi(z_{t-1} | a) \phi(tc_t - \sum_{k=1}^{t-1} z_k | a) dz_{t-1} \dots dz_1}{\int_{-\infty}^{\infty} g(a) \left[\int_{c_1}^{\infty} \phi(z_1 | a) \cdots \int_{(t-1)c_t - \sum_{k=1}^{t-2} z_k}^{\infty} \phi(z_{t-1} | a) \phi(tc_t - \sum_{k=1}^{t-1} z_k | a) dz_{t-1} \dots dz_1 \right] da}.$$

The expected ability of a marginal survivor is given by

$$\bar{e}^M[A | c, t] = \int_{-\infty}^{\infty} a \bar{g}(a | c, t) da.$$

At each a , the probability of survival, $\bar{\mathcal{S}}_t^M(a, c)$, is decreasing with t , and bounded below by zero. Hence the sequence converges at each a . Let

$$\bar{\mathcal{S}}^M(a, c) = \lim_{t \rightarrow \infty} \bar{\mathcal{S}}_t^M(a, c) \text{ for each } a \in \mathbb{R}.$$

In a hierarchy with memory, the *limit distribution of surviving risk takers* has density defined by

$$\frac{g(a) \bar{\mathcal{S}}^M(a, c)}{\int_{-\infty}^{\infty} g(a) \bar{\mathcal{S}}^M(a, c) da}. \quad (4)$$

I show in [Lemma 3](#) that the support is $[\bar{c}, \infty)$, where $\bar{c} = \limsup c_t$.

Because the survival function $\bar{\mathcal{S}}_t^M(\cdot, c)$ is increasing, the limit function $\bar{\mathcal{S}}^M(\cdot, c)$ is non-decreasing.

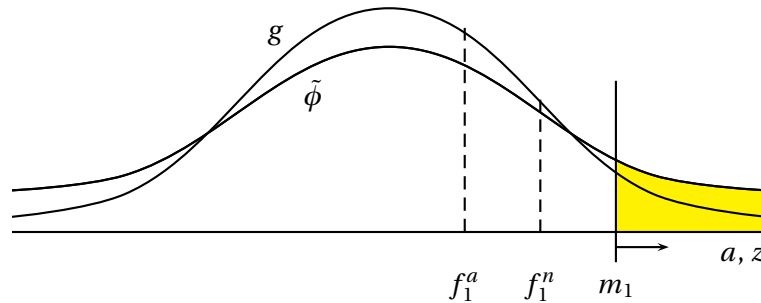


FIGURE 1. First stage of the hierarchy

3. ANTIDISCRIMINATION GOALS: AN IMPOSSIBILITY RESULT

Before turning to how promotion plays out in a hierarchy, I use a single round of promotion to show how differences in risk taking create a conflict among the following three natural objectives of labor policy:

- gender-blind promotion standards
- equal survival rates
- equal abilities of survivors.

In [Figure 1](#), the distribution of abilities A (hence the distribution of signals generated by females) is shown by the density g . For this purpose, suppose that [Assumption A](#) holds. Then the distribution of the signal Z generated by males is a mean-preserving spread of g . The marginal distribution of Z is represented by the density $\tilde{\phi}$ in [Figure 1](#).

Suppose the males' promotion standard at stage 1 is m_1 . The resulting probability of survival is the area to the right of m_1 . If the promotion standard is gender blind, all females with signal (ability) above m_1 are also promoted. However, there are fewer females than males, provided $m_1 > 0$, since $\tilde{\phi}$ is a mean-preserving spread of g .

At the same time, females who exceed the standard m_1 have higher average ability than males. This follows from a very general result, stated in [Lemma 1](#) below, that the average ability of surviving males is lower than the average ability of surviving females in any period where males have a higher survival rate. We thus see immediately—and it is stated formally in [Proposition 1](#)—that gender-blind standards are incompatible with equal promotion rates and with equal abilities. To equalize the survival rates, the female promotion standard must be lower, namely, f_1^n in [Figure 1](#), and to equalize abilities, the female promotion standard must be lower still, namely f_1^a .

That the promotion standards are ordered as drawn is stated in [Proposition 1](#).

LEMMA 1 (If males have a higher survival rate than females, males have lower average ability). *Let (m, f) be promotion standards for males and females. Suppose that the*

survival rate of males is no smaller than the survival rate of females at time t :

$$\begin{aligned} E[S_t^M(A, m)] &\geq E[S_t^F(A, f)] \quad (\text{without memory}) \\ E[\tilde{S}_t^M(A, m)] &\geq E[S_t^F(A, f)] \quad (\text{with memory}). \end{aligned}$$

Then the expected ability of a random surviving male is smaller than the expected ability of a random surviving female:

$$\begin{aligned} E^M[A | m, t] &< E^F[A | f, t] \quad (\text{without memory}) \\ \tilde{E}^M[A | m, t] &< E^F[A | f, t] \quad (\text{with memory}). \end{aligned}$$

The lemma is proved in the [Appendix](#). It uses essentially no assumptions on the distributions or promotion standards, and is thus very general. The intuition is simply that, because males generate noisy signals, mistakes are made in promoting them. Some high ability males are not promoted, and some low ability males are promoted. In contrast, no mistakes are made in promoting females. Hence, if males and females are promoted in equal numbers, the average ability of the surviving males must be lower.

PROPOSITION 1 (The impossibility of equalizing survival rates, abilities and standards). *Let m be the promotion standards for males. Let f^n (respectively f^a) be promotion standards for females such that their survival rate (respectively, ability) equals that of surviving males at stage 1. Then*

- (i) $f_1^a < f_1^n$
- (ii) if [Assumption A](#) holds and $m_1 > 0$, then $f_1^n < m_1$.

PROOF. Part (i) follows because $E[S_1^F(A, f^n)] = E[S_1^M(A, m)] < E[S_1^F(A, f^a)]$. The first equality is by definition of f^n and the second follows from [Lemma 1](#). If $E[S_1^M(A, m)] \geq E[S_1^F(A, f^a)]$ then the expected ability of male survivors would be smaller than that of females at stage 1, a contradiction.

(ii) Using [Assumption A](#), the distribution of ability second-order dominates the distribution of the signal Z . Both have mean 0, so for $m_1 > 0$, the probability that a male survives (namely, the probability that $Z > m_1$) exceeds the probability that a female survives according to the same standard (the probability that $A > m_1$). Hence, to increase the probability that a female survives, and in particular to equalize the survival rates, we must have $f_1^n < m_1$. $\square$

Proposition 1 illustrates why “affirmative action” in labor markets is a vexed issue. It is not possible at the same time to equalize numbers, equalize abilities, and also to satisfy the procedural objective of having gender-blind standards.²

²[Spitzer \(1979\)](#) makes a similar argument, showing how courts and agencies must balance objectives that can be mutually inconsistent. Most pertinently, he applies his analysis to the *Bakke* decision (*University of California v. Bakke*, 438 U.S. 265 (1978)), in which the split decision of the Supreme Court is largely due to the fact that different justices were concerned with different objectives, all legitimate, which cannot simultaneously be satisfied.

However, employment law seems to have all three objectives. Antidiscrimination law in the U.S. dates mainly to Title VII of the 1964 Civil Rights Act, and its extension in the 1972 Equal Employment Opportunity Act.³ These Acts disallow disparate *treatment* by employers based on race, color, sex, national origin, or religion, with respect to hiring, promotion, compensation, or termination decisions.⁴ At the same time, an employer's practices might run afoul of Title VII if they have a disparate *impact* on a protected group.⁵

Whereas disparate treatment concerns the individual, and in particular, whether the employer intended to discriminate against the individual, disparate impact is about the effects of an employment practice on an entire group. The employer's intent is not at issue in disparate impact cases. A Title VII disparate-impact violation results when a seemingly neutral employment practice affects opportunities in ways that are not race- or gender-neutral, and where the employer cannot prove that the practice is justified under Title VII's business necessity defense.⁶

Proposition 1 points out that the bans on disparate treatment and disparate impact are contradictory. Title VII insists on gender-blind employment practices, but gender-blind employment practices lead to disparate impacts on the numbers promoted and on survivors' abilities. These disparate impacts cannot be remedied by gender-blind promotion standards. Under the conditions of **Proposition 1**, the standards for females must be lower than for males early in the hierarchy, but (as we will see) higher as the hierarchy progresses.

The contradiction between gender- or race-blindness and disparate impact surfaces in personnel policies that seem incomprehensible.⁷ MIT's Human Resources page⁸ poses the question "Is affirmative action an employment quota system?" Since "quotas

³A great deal has been written on the legal details and their impact. On the economic side, see **Rodgers (2006)**. The chapter **Holzer and Neumark (2006)** outlines the broad contours of the law. For a history of how the law developed, see **Blumrosen (1993)**, and for detail on the evidentiary conundrums that arise, see, for example, **Krieger (1995)**.

⁴The more pro-active notion of "affirmative action" originates in Executive Orders 10925 (1961), 11246 (1965, 1967), and 11625 (1971) of the Kennedy, Johnson, and Nixon administrations, respectively, aimed at federal contractors.

⁵The ban on disparate impacts evolved from policies of the Equal Employment Opportunity Commission (EEOC), which was created to enforce the Civil Rights Act. See **Blumrosen (1993, 73–75)**, and **Leiter and Leiter (2002, 43–47)**. In the absence of clear definitions or a clear legislative intent behind the Civil Rights Act, the EEOC pursued policies that imputed to the law an ambition to improve the economic circumstances of minorities. This led to policies against disparate impact that were enunciated as law by the Supreme Court in *Griggs v. Duke Power Co.*, 401 U.S. 424 (1971).

⁶**Krieger (1995)** discusses why it is difficult to prove discrimination under both theories. She argues that disparate treatment may result from subtle stereotypes even where there is no discriminatory intent. Where there are patterns and practices of discrimination, also known as systemic disparate treatment, she argues that disparate impact theory is like a software "patch," (p. 1227). Although disparate impact theory circumvents the intent requirement, it creates problems that make judges reluctant to apply it. Moreover, Krieger argues that, in response to disparate impact theory, employers have shifted from quantifiable criteria like test scores to subjective standards involving unquantifiable variables.

⁷That contradictions arise from competing values has been convincingly discussed in the case of race by **Edley (1996)**.

⁸http://web.mit.edu/hr/aa/what_is.html

are expressly forbidden by federal law,” the answer is that if “underutilization” of minority groups is discovered, ... “placement goals are established ... Goals are targets, not quotas.” The page does not explain the difference between targets and quotas, but asserts that there is no conflict between targets and “principles of merit.” **Proposition 1** suggests otherwise.

4. THE HIERARCHIES

I now turn to what happens as the hierarchy continues after the first stage. In particular, I assume there are infinitely many periods of promotion. The infinite sequence is unrealistic, but reveals a pattern for late stages that might otherwise be obscure. In particular, with gender-blind standards, there is a reversal in the relative numbers and relative abilities of the surviving populations of males and females. At the beginning of the hierarchy, under the reasonable conditions of **Figure 1**, gender-blind strategies lead to a higher survival rate among males than among females, but lower ability. At the end, this is reversed. I show that the reversal occurs in hierarchies with and without memory.

4.1 *The hierarchy without memory*

I first discuss how the hierarchy without memory plays out. **Proposition 2** says that, in a hierarchy with gender blind standards, there is eventually a higher survival rate among females, but they have lower ability. This is essentially because males have an opportunity to throw themselves out of the pool at every stage, and will eventually do so, except possibly those with very high ability.

Lemma 2 is proved in the **Appendix**.

LEMMA 2 (Only males with very high ability survive in the limit). *Let $m = m_1, m_2, \dots, m_t, \dots$ be bounded promotion standards for a hierarchy without memory.*

- (i) *Given $\varepsilon > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the survival rate (1) of males at stage t is less than ε .*
- (ii) *Given $\zeta \in (0, 1)$ and $x > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the probability is smaller than ζ that a male survivor has ability less than x .*
- (iii) *Given $x > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the expected ability (2) of a male survivor is larger than x .*
- (iv) *Given $x > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the expected ability (3) of a marginal male survivor is larger than x .*

PROPOSITION 2 (Gender-blind promotions without memory). *Let $c = c_1, c_2, \dots, c_t, \dots$ be bounded, gender-blind promotion standards for a hierarchy without memory. Then there exists $\tilde{t}$ such that for $t > \tilde{t}$, the survival rate of males is smaller than the survival rate of females, but the surviving males have higher expected ability.*

The proposition follows from **Lemma 2**. That males eventually survive with smaller probability than females follows directly from **Lemma 2(i)**, by choosing $\varepsilon > 0$ such that $(1 - G(c_t)) > \varepsilon$ for all t . That male survivors eventually have higher expected ability than surviving females follows by choosing x in **Lemma 2(iii)** to be larger than the (bounded) expected ability of surviving females.

Now suppose that the survival rates are equal rather than that standards are gender blind. Then **Proposition 3** says that the males' promotion standards cannot be bounded below. This can be interpreted to mean that promotion standards become irrelevant; all surviving males are eventually promoted.

Since the promotion standards for females are bounded, there will eventually be a stationary, positive measure of surviving females. Hence, the population of surviving males must also become stationary with a positive measure of survivors. But if the males' promotion standards are bounded below, males still have an opportunity to throw themselves out of the pool at every stage, and their numbers will decline. To avoid this, the promotion standards must become very low in order to keep all the males in the pool. Males appear to be getting special treatment, but no males are profiting from the special treatment because their performance is well above the very low standard.

PROPOSITION 3 (Equal survival rates in a hierarchy without memory). *Let (m, f) be promotion standards in a hierarchy without memory, such that males and females have the same survival rates at each stage t . If the promotion standards f are bounded, then the performance standards m are not bounded below.*

PROOF. Since $f_1, f_2, \dots, f_t, \dots$ converge to a finite number, the female survival rates $E[S_t^F(A, f)]$, $t = 1, 2, \dots$, converge to a positive number, say $L > 0$, and, by hypothesis, the male survival rates $E[S_t^M(A, m)]$, $t = 1, 2, \dots$, converge to the same limit, L . Suppose, contrary to the proposition, that the sequence m is bounded below by $\underline{m}$. Since $(1 - \Phi(m_t | a)) \leq (1 - \Phi(\underline{m} | a))$ for all a , the male survival rate at stage t satisfies

$$E[S_t^M(A, m)] = \int_{-\infty}^{\infty} g(a) S_t^M(a, m) da \leq \int_{-\infty}^{\infty} g(a) (1 - \Phi(\underline{m} | a))^t da. \quad (5)$$

Choose an $\varepsilon > 0$ such that $\varepsilon < L$. Choose $\tilde{a}, \hat{a}$ such that $\hat{a} < \tilde{a}$ and

$$\begin{aligned} 1 - G(\tilde{a}) &< \varepsilon/3 \\ G(\hat{a}) &< \varepsilon/3. \end{aligned}$$

Due to the monotone likelihood ratio property, $\Phi(\underline{m} | a)$ decreases with a . Choose $\hat{t}$ such that $(1 - \Phi(\underline{m} | \tilde{a}))^{\hat{t}} < \varepsilon/3$. Then if $t > \hat{t}$, the upper bound on the male survival rate (5) at stage t , satisfies

$$\begin{aligned} &\int_{-\infty}^{\hat{a}} g(a) (1 - \Phi(\underline{m} | a))^t da + \int_{\hat{a}}^{\tilde{a}} g(a) (1 - \Phi(\underline{m} | a))^t da + \int_{\tilde{a}}^{\infty} g(a) (1 - \Phi(\underline{m} | a))^t da \\ &< \int_{-\infty}^{\hat{a}} g(a) da + [G(\tilde{a}) - G(\hat{a})] (1 - \Phi(\underline{m} | \tilde{a}))^t + \int_{\tilde{a}}^{\infty} g(a) da \\ &< \varepsilon/3 + (1 - \Phi(\underline{m} | \tilde{a}))^t + \varepsilon/3 < \varepsilon < L. \end{aligned}$$

This is a contradiction. □

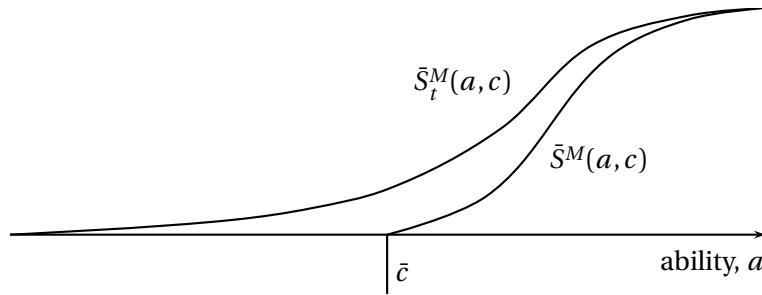


FIGURE 2. Survival in a Hierarchy with Memory

PROPOSITION 4 (Equal ability in a hierarchy without memory). *(i) Suppose, in a hierarchy without memory, that the expected abilities of surviving males and females are equal at stage $\hat{t}$ under the promotion standards (m, f) . Then the female survival rate at stage $\hat{t}$ is greater than the male survival rate.*

(ii) In a hierarchy without memory, there are no bounded promotion standards (m, f) for which surviving males have the same average ability as surviving females at each stage t .

Proposition 4(i) follows directly from **Lemma 1**, which would otherwise be contradicted. **Proposition 4**(ii) follows from **Lemma 2**(iii) by choosing x larger than the expected ability of female survivors at every t .

4.2 The hierarchy with memory

I now show that, if the hierarchy has memory, males survive in the limit with positive probability. As the hierarchy progresses, it is increasingly difficult to overturn a survivor's accumulated success. Each new signal is blended into the average, and its effect is therefore muted. Nevertheless, if the standards are gender blind, it is still true that eventually a smaller proportion of males than females survives, and they have higher average ability. This must be proved in a different manner than the analogous result without memory (**Proposition 2**), since the analog to **Lemma 2** does not hold.

Proposition 5 follows from the shape of the limiting survival function $\bar{S}^M(\cdot, c)$, described in **Lemma 3** and shown in **Figure 2**. The proof is in the **Appendix**.

LEMMA 3 (The limit distribution in a hierarchy with memory). *Suppose that the promotion standards are $c = c_1, c_2, \dots, c_t, \dots$, and $\limsup c_t = \bar{c}$. For each $a \leq \bar{c}$, $\bar{S}^M(a, c) = 0$. Given $\varepsilon > 0$, for each $a > \bar{c} + \varepsilon$, $\bar{S}^M(a, c) > 0$.*

Proposition 5 uses the hypothesis that promotion standards are nondecreasing. With decreasing promotion standards, all females with ability above c_1 would survive, and none would be eliminated after stage one. This is not true of males. If the performance standards decrease rapidly, males with abilities lower than c_1 might survive in

large numbers, and the limiting expected ability of males could be lower than that of females. In any case, increasing promotion standards are the more natural case.

PROPOSITION 5 (Gender-blind promotions with memory). *Let $c = c_1, c_2, \dots, c_t, \dots$ be non-decreasing, gender-blind promotion standards that converge to $\bar{c}$ in a hierarchy with memory. Then there exists $\hat{t}$ such that for $t > \hat{t}$, the survival rate of males is smaller than that of females, but the surviving males have higher expected ability.*

PROOF. Using **Lemma 3**, for $a < \bar{c}$, neither males nor females survive. For $a > \bar{c}$, the probability that a female survives is one, while for males, the survival probability is less than one: $\bar{S}^M(a, c) < 1 - \Phi(c_1 | a) < 1$. Thus, the limit survival rate of males is smaller than that of females.

Because the males' limit probabilities of survival are nondecreasing with a , and strictly increasing on some domain (since $\bar{S}^M(a, c) > \bar{S}^M(\bar{c}, c)$ for some $a > \bar{c}$), the limit distribution of their abilities first-order dominates the limit distribution of females' abilities. Thus, the expected ability of surviving males is greater than that of surviving females. $\square$

PROPOSITION 6 (Equal average ability with memory). *Suppose the hierarchy has memory, and suppose that (m, f) are promotion standards such that the expected abilities of a random surviving male and a random surviving female are equal at stage t . Then the survival rate of females at stage t is greater than that of males.*

This result follows directly from **Lemma 1**.

5. PRIVATE OBJECTIVE FUNCTIONS

So far, the strategy of this paper has been to study the promotion standards that satisfy some social objective, such as equal survival rates of males and females or gender-blind standards, and to show the conflicts that are created. I now characterize the promotion standards that arise without legal constraints. I do not start from the premise that employers dislike diversity, or that they are willing to sacrifice profit in order to avoid it, but rather from the premise that employers are profit maximizers whose decisions may nevertheless lead to disparate impacts. The objective is to see which of the social goals are preserved, if any, and whether "discrimination" can be identified.⁹

The implicit assumption in what follows is that wages can be different at different stages of the hierarchy, but wages cannot be different for males and females, and wages do not depend directly on signals.

5.1 Promotions across firms

Suppose that, to be promoted, an agent must get a job with a different employer. An employer hiring at t selects from the survivors to $t - 1$. This creates externalities among

⁹The conceit in this paper is that promotion standards are set according to some well-conceived objective, either constrained by law, or guided by profit, but not guided by reference to the pool that is already promoted. For an interesting discussion of the latter, see **Sobel (2001)**.

firms, since the distribution of abilities in an employer's pool is determined by previous employers' promotion decisions.

Because an employer retains its workers for a single period, its sole objective is to take the workers with highest ability. For each $t = 1, 2, \dots$, let W_t be the profit objective when the industry does not have memory, and let $\bar{W}_t$ be the objective when the industry has memory:

$$W_t(m, f) = \int_{-\infty}^{\infty} (a - w_t) S_t^M(a, m) g(a) da + \int_{f_t}^{\infty} (a - w_t) g(a) da \quad (6a)$$

$$\bar{W}_t(m, f) = \int_{-\infty}^{\infty} (a - w_t) \bar{S}_t^M(a, m) g(a) da + \int_{f_t}^{\infty} (a - w_t) g(a) da. \quad (6b)$$

The profit function is an aggregate for all employers at stage t . The wage rate is w_t , and ability a is understood as the profit of employing an agent with ability a . I assume that the wages are bounded and nondecreasing, and converge to $\bar{w}$. This implies that the female promotion standards f are bounded and nondecreasing, and also converge.¹⁰

It is straightforward to show that the optimal standards equalize the expected abilities of marginal survivors.

LEMMA 4. *If the promotion standards (m, f) maximize total ability net of wages, (6a) (respectively, (6b)), then the standards (m_t, f_t) equalize the abilities of marginal males and marginal females at t . In particular, (7a) (respectively, (7b)) holds.*

$$e^M[A | m, t] - w_t = f_t - w_t = 0 \quad (7a)$$

$$\bar{e}^M[A | m, t] - w_t = f_t - w_t = 0. \quad (7b)$$

Proposition 7 characterizes the standards that arise endogenously when promotions are across firms. Males are held to a higher standard at the earliest stage provided fewer than half are promoted, but in the hierarchy without memory, males are eventually promoted on weak performance, much weaker than required of females. This is for a reason similar to when standards are chosen to equalize promotion rates (**Proposition 3**). If the males' promotion standards did not become low, males would keep throwing themselves out of the pool, and the expected ability of the marginal (and average) survivor would keep rising. But if the objective is to stop them from getting thrown out of the pool, and hence to equalize the abilities of marginal survivors, then the males' standard must become low. A male's probability of generating such a low standard must go to zero.

For the hierarchy with memory, I use **Assumption B** in order to avoid complicated arguments based on rates of convergence.

ASSUMPTION B. In the limit of the hierarchy with memory, each male's ability is known, and the distribution of males' abilities has the limit distribution (4).

¹⁰In (6a) and (6b), that is why I write the objective function for females with a minimum ability f_t rather than $\max_{d \leq t} \{f_d\}$.

Since the abilities of males are revealed in the limit, their standard for promotion is ultimately the same as for females. There are marginal females in the limit, in the sense that there remain a positive mass of females with abilities in a neighborhood of the limit promotion standard, $\tilde{f}$. However, due to the selection process before reaching the limit, there are no marginal males. **Proposition 7**(ii) and (iii) say that the probability of a risk taker being marginal at late stages of the hierarchy converges to zero.

In the hierarchy with memory, the marginal probability density of the signal Z_t at stage t is given by

$$\tilde{\phi}_t(z) = \int_{-\infty}^{\infty} \phi(z | a) \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} da.$$

PROPOSITION 7 (Promotion standards that equalize the abilities of marginal survivors). *Suppose that at each stage t , the promotion standards (m, f) maximize total ability net of wages (6a) (respectively, (6b)), so that marginal male and female survivors have equal expected abilities. Suppose **Assumption A** holds.*

- (i) *Whether or not the hierarchy has memory, if $m_1 > 0$, females face a lower promotion standard than do males at stage 1. That is, $m_1 > f_1$.*
- (ii) *Suppose the hierarchy does not have memory. Then males eventually face lower promotion standards than females, but there are almost no marginal males. That is, given $\varepsilon > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, $m_t < f_t$ and $\tilde{\phi}_t(m_t) < \varepsilon$.*
- (iii) *Suppose the hierarchy has memory, and that **Assumption B** holds. Then males eventually face the same promotion standards as females, but there are almost no marginal males. In particular, $\limsup m_t = \limsup f_t$. Given $\varepsilon > 0$, there exists $\tilde{t}$ and $\delta > 0$ such that for $t > \tilde{t}$, $\int_{-\infty}^{\tilde{m}+\delta} g(a) \tilde{S}_t^M(a, m) da < \varepsilon$.*

PROOF. (i) It is enough to show that the expected ability of a marginal male is less than his promotion standard m_1 . The expected ability of a marginal male can be written

$$\begin{aligned} e^M[A | m, 1] &= \int_{-\infty}^{\infty} a \frac{g(a) \phi^u(m_1 - a)}{\int_{-\infty}^{\infty} g(a) \phi^u(m_1 - a) da} da \\ &= \int_{-\infty}^{\infty} (m_1 - x) \frac{g(m_1 - x) \phi^u(x)}{\int_{-\infty}^{\infty} g(a) \phi^u(m_1 - a) da} dx \\ &= m_1 - \int_{-\infty}^{\infty} x \frac{g(m_1 - x) \phi^u(x)}{\int_{-\infty}^{\infty} g(a) \phi^u(m_1 - a) da} dx < m_1. \end{aligned} \quad (8)$$

The inequality follows because the integral in (8) is positive. The denominator is

positive, and the numerator can be written

$$\begin{aligned} & \int_0^\infty x g(m_1 - x) \phi^u(x) dx + \int_{-\infty}^0 x g(m_1 - x) \phi^u(x) dx \\ &= \int_0^\infty y g(m_1 - y) \phi^u(y) dy - \int_0^\infty y g(m_1 + y) \phi^u(-y) dy \\ &= \int_0^\infty y [g(m_1 - y) - g(m_1 + y)] \phi^u(y) dy > 0. \end{aligned}$$

In the second line, $\phi^u(y) = \phi^u(-y)$ due to the symmetry of ϕ^u and in the last line, $[g(m_1 - y) - g(m_1 + y)] > 0$ due to $m_1 > 0$ and the symmetry and single-peakedness of g .

(ii) To show that $m_t < f_t$, let $\underline{f}$ and $\bar{f}$ be the lower and upper bounds for f . Let $x \geq \bar{f}$. Using Lemma 2(iv), if m is bounded, then for large t , the expected ability of the marginal male survivor is larger than x , hence larger than f_t , a contradiction. Therefore m is not bounded, and in particular, is not bounded below. Let $\bar{t}$ be large enough that for $t > \bar{t}$, $m_t < \underline{f} \leq f_t$.

I now show that $\tilde{\phi}_t(m_t) < \varepsilon$ for $t > \bar{t}$. Using Assumption A, an upper bound for $\phi(m_t | a) = \phi^u(m_t - a)$ is $\phi^u(0)$. Let $\zeta, \gamma > 0$ be such that $\zeta \phi^u(0) + \gamma < \varepsilon$.

Choose $x > 0$ and $\bar{t}$ large enough so that $\phi^u(m_t - a) < \phi^u(\bar{f} - a) < \gamma$ for $a > x$ and, using Lemma 2(ii), so that

$$\int_{-\infty}^x \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da < \zeta.$$

Then

$$\begin{aligned} \tilde{\phi}_t(m_t) &= \int_{-\infty}^x \phi(m_t | a) \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da \\ &\quad + \int_x^\infty \phi(m_t | a) \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da \\ &< \int_{-\infty}^x \phi(m_t | a) \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da + \gamma \int_x^\infty \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da \\ &< \phi^u(0) \int_{-\infty}^x \frac{g(a) S_t^M(a, m)}{\int_{-\infty}^\infty g(a) S_t^M(a, m) da} da + \gamma < \phi^u(0) \zeta + \gamma < \varepsilon. \end{aligned}$$

(iii) The profit

$$\int_{\bar{m}}^\infty (a - \bar{w}) \bar{S}^M(a, m) g(a) da$$

is maximized by choosing $\bar{m} = \bar{w}$, hence $\limsup m_t$ should be $\bar{m}$. Since $\limsup f_t = \bar{f} = \bar{w}$, the result follows. That

$$\int_{-\infty}^{\bar{m}+\varepsilon} g(a) \bar{S}_t^M(a, m) da < \varepsilon$$

for large t follows from Lemma 3 and Figure 2. $\square$

When the promotion standards equalize the expected abilities of marginal survivors, both the survival rates and abilities of male and female survivors differ. These differences have an efficiency explanation, and in particular do not follow from gender preferences on the part of employers.

However, disparate treatment due to a legitimate business objective is hard to distinguish from disparate treatment due to gender preferences.

Suppose, for example, that firms choose promotion standards according to the following “discriminatory” objective:

$$\max_{\{m_t, f_t\}} W_t(m, f) + U \left(\frac{E[S_t^M(A, m)]}{E[S_t^F(A, f)]} \right).$$

If U is increasing, the objective explicitly favors males. But the standard for males may still be higher at the beginning of the hierarchy than for females, and lower at the end, whether the objective is W or $W + U$. Because of this ambiguity, courts and policymakers may resort to procedural rules such as a requirement for gender-blind standards or equal survival rates.

5.2 Promotions within the firm: preserving options

Suppose now that all promotions take place within a single firm. Then the firm itself profits from the information it collects through its promotion decisions. I show that the firm has an incentive to keep a male in the pool at early stages, despite a moderately bad signal, in order to collect more information and preserve an option. In particular, I show that at an early stage the firm retains males with lower expected ability than is tolerated in promoted females. In this sense, males are favored.

Consider a hierarchy with three stages ($t = 0, 1, 2$) and two promotions (at $t = 1, 2$). At stage 0, there is no employer, but agents generate signals of ability. These are used for hiring in the first promotion stage. A firm's objective function is equal to the sum of workers' abilities in each promotion period, net of wages.

The firm's objective function is

$$\begin{aligned} \Pi(f_1, f_2, m_1, m_2) = & \int_{f_1}^{\infty} (a - w_1)g(a) da + \int_{\max\{f_1, f_2\}}^{\infty} (a - w_2)g(a) da \\ & + \int_{-\infty}^{\infty} (a - w_1)g(a)[1 - \Phi(m_1 | a)] da \\ & + \int_{-\infty}^{\infty} (a - w_2)g(a)[1 - \Phi(m_1 | a)][1 - \Phi(m_2 | a)] da. \quad (9) \end{aligned}$$

In Proposition 8(i), I assume that wages in both periods are equal. This assumption isolates the incentive to preserve options from the problem that the cost of a mistake is compounded when wages are increasing.

PROPOSITION 8 (Preserving options within the firm). *Suppose there are three periods with two promotions that take place within a single firm. Suppose the promotion standards $(f_1, f_2), (m_1, m_2)$ maximize (9).*

- (i) *Suppose that $w_1 = w_2 = w$. Then the expected ability of a marginal male at the first promotion stage is smaller than the expected ability of a marginal female. That is, $e^M[A | m, 1] < f_1$.*
- (ii) *The expected ability of a marginal male at the final promotion stage is the same as that of a marginal female. That is, $e^M[A | m, 2] = f_2$.*

PROOF. Since wages are nondecreasing, the optimizer of (9) satisfies

$$f_1 = w_1 \quad (10)$$

$$f_2 = w_2 \quad (11)$$

$$\int_{-\infty}^{\infty} (a - w_1)g(a)\phi(m_1 | a)da + \int_{-\infty}^{\infty} (a - w_2)g(a)\phi(m_1 | a)[1 - \Phi(m_2 | a)]da = 0 \quad (12)$$

$$\int_{-\infty}^{\infty} (a - w_2)g(a)[1 - \Phi(m_1 | a)]\phi(m_2 | a)da = 0. \quad (13)$$

Write $\mathcal{M}_1$ for the probability of being a marginal male at the first promotion:

$$\mathcal{M}_1 = \int_{-\infty}^{\infty} g(a)\phi(m_1 | a)da = \int_{-\infty}^{\infty} g(\alpha + w)\phi(m_1 | \alpha + w)d\alpha.$$

Write $\mathcal{M}_2$ for the probability of being a marginal male in the first promotion and also surviving the second promotion:

$$\begin{aligned} \mathcal{M}_2 &= \int_{-\infty}^{\infty} g(a)\phi(m_1 | a)[1 - \Phi(m_2 | a)]da \\ &= \int_{-\infty}^{\infty} g(\alpha + w)\phi(m_1 | \alpha + w)[1 - \Phi(m_2 | \alpha + w)]d\alpha. \end{aligned}$$

Then the first-order condition (12) can be written

$$\begin{aligned} \mathcal{M}_1 \int_{-\infty}^{\infty} \alpha \frac{g(\alpha + w)\phi(m_1 | \alpha + w)}{\mathcal{M}_1} d\alpha \\ + \mathcal{M}_2 \int_{-\infty}^{\infty} \alpha \frac{g(\alpha + w)\phi(m_1 | \alpha + w)[1 - \Phi(m_2 | \alpha + w)]}{\mathcal{M}_2} d\alpha = 0. \end{aligned} \quad (14)$$

The density in the first integral of (14) describes the males who were marginal at the first promotion. The density in the second integral of (14) describes the males remaining from that group after the second promotion. Because $\Phi(m_2 | \alpha + w)$ is decreasing with

a , the distribution in the second integral stochastically dominates the distribution in the first integral. Because of stochastic dominance, the expected ability represented by the first integral is smaller than the expected ability represented by the second integral. Since the two terms sum to zero, the first integral is negative and the second is positive. That is,

$$\int_{-\infty}^{\infty} (a - w) \frac{g(a)\phi(m_1 | a)}{\int_{-\infty}^{\infty} g(a)\phi(m_1 | a) da} da < 0 < \int_{-\infty}^{\infty} (a - w) \frac{g(a)\phi(m_2 | a)[1 - \Phi(m_2 | a)]}{\int_{-\infty}^{\infty} g(a)\phi(m_2 | a)[1 - \Phi(m_2 | a)] da} da.$$

Using (10), the first inequality implies $e^M[A | m, 1] < w = f_1$. This proves (i).

Part (ii) follows from (11), which says $f_2 = w_2$, and from (13), which says $e^M[A | m, 2] = w_2$. $\square$

When promotions are within the firm, one cannot say in general whether males are promoted at a lower or higher standard than females at the first stage, but conditional on a given standard for females, more males are promoted within the firm than if promotions were across firms. Again, this is because the firm internalizes the information it generates by keeping males in the pool.

Finally, **Proposition 8** also describes the optimal policy when the firm's profit depends on the signals produced by the workers rather than on their abilities. For example, the value produced in an academic labor market is the quality of the papers written, rather than the ability of the professor who produced them. The objective function would then be

$$\begin{aligned} \Pi^z(f_1, f_2, m_1, m_2) = & \int_{f_1}^{\infty} (a - w_1)g(a)da + \int_{f_2}^{\infty} (a - w_1)g(a)da \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (z_1 - w_1)g(a)[1 - \Phi(m_1 | a)]\phi(z_1 | a)dz_1 da \\ & + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (z_2 - w_1)g(a)[1 - \Phi(m_1 | a)][1 - \Phi(m_2 | a)]\phi(z_2 | a)dz_2 da \end{aligned}$$

But, integrating on z_1 and z_2 , since $\int_{-\infty}^{\infty} z\phi(z | a)dz = a$, we have $\Pi^z(f_1, f_2, m_1, m_2) = \Pi(f_1, f_2, m_1, m_2)$.

6. CONCLUSION

Affirmative action policies have been justified and evaluated from many different perspectives, on both efficiency grounds and equity grounds. For the most part, economists have focussed on efficiency, especially productive efficiency. For example, **Holzer and Neumark (2000, 559)** argue from an extensive empirical literature that “affirmative

action offers significant redistribution toward women and minorities, with relatively small efficiency consequences.” Among the ingenious theoretical arguments for why affirmative action policies enhance efficiency are those of [Lundberg and Startz \(1983\)](#) and [Lundberg \(1991\)](#), who consider a model of statistical discrimination where wages depend on imperfect signals of ability. They show, among other things, that the pooling of workers with different signaling abilities creates more incentive to invest in human capital. [Milgrom and Oster \(1987\)](#) argue that affirmative action policies can efficiently prevent employers from underpromoting women and minorities. The incentive to underpromote derives from a fear of revealing the worth of their employees to rival firms, a threat that is higher for the more “invisible” workers, such as women and minorities.

I have not made efficiency central to my analysis, since it is unclear how to define it in the context studied here. I have taken the social objectives of gender blindness or equal promotions as primitive, rather than deriving those objectives from a welfare function. This has allowed me to illuminate conflicts among those objectives that derive from gender differences in behavior, in particular risk taking. I have simply assumed that males are hard-wired for risk taking and females are not, and that all agents want to stay in the hierarchy if allowed.

The efficiency criterion that springs to mind for most economists is that marginal survivors should have the same expected ability. This is the criterion that drives promotion standards when promotions are across firms, but only because the employers do not internalize the benefits of collecting information on risk takers, which can possibly create a better pool for the long run. When promotions are within firms, the firm has an incentive to preserve an option on males at the beginning, since more information will be collected later. The firm itself profits from this information. Thus, the promotion standards are such that the marginal male has lower expected ability than the marginal female, and in this sense, males are favored. However, the promotion standard itself may be higher for males than for females in both cases.

There are many other gender differences at work as well. Some of these differences can be confounding factors that overturn any intrinsic difference in risk taking. In particular, [Becker and Eagly \(2004\)](#) find that females were at least as likely as males to put themselves at risk in protecting Jews in the Holocaust, and females are considerably more likely to put themselves at risk by donating kidneys to relatives in need. However, the authors hypothesize that such behavior might be rooted in a greater willingness of females to care for others, or to heed an ethical calling. Females may be motivated by objectives that overcome, and therefore obscure, an aversion to risk.

Further, as I mentioned in the introduction, the noisiness of the signals is not entirely under the control of the employee. Even if males are more risk-taking, they may be more closely observed within each promotion period than females. The latter hypothesis is consonant with the [Milgrom and Oster \(1987\)](#) “invisibility” hypothesis. If that effect is strong enough, then females should be understood as the workers who present noisy signals, and the males should be understood as generating more accurate signals.

In any case, the main point of the paper is that gender differences in tastes set up a conflict between procedural fairness and disparate impacts. Differences in risk taking are one example of that larger point.¹¹ A legitimate business purpose is a defense to disparate impact, but as mentioned in Section 5, a legitimate business purpose cannot always be distinguished from discriminatory intent.

APPENDIX

PROOF OF LEMMA 1. The same argument applies whether the survival rates are calculated with or without memory. Arguing for the hierarchy without memory, we have

$$\begin{aligned}
 & \int_{-\infty}^{\infty} a \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da \\
 &= \int_{-\infty}^{\infty} (a - f_t) \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da \\
 &= \int_{-\infty}^{f_t} (a - f_t) \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da \\
 &\quad + \int_{f_t}^{\infty} (a - f_t) \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da \\
 &< 0.
 \end{aligned}$$

In the third line, where $a - f_t < 0$, we have $S_t^F(a, f) = 0$ and $0 < S_t^M(a, m) < 1$. Therefore, $S_t^M(a, m)/E[S_t^M(A, m)] - S_t^F(a, m)/E[S_t^F(A, f)] > 0$, so the integrand is negative. In the fourth line, where $a - f_t \geq 0$, we have $0 < S_t^M(a, m) < 1 = S_t^F(a, f)$. Since $E[S_t^M(A, m)] \geq E[S_t^F(A, f)]$, we have $S_t^M(a, m)/E[S_t^M(A, m)] < S_t^F(a, f)/E[S_t^F(A, f)]$. Therefore the integrand is again negative. Then

$$\begin{aligned}
 0 &> \int_{-\infty}^{\infty} (a - f_t) \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da \\
 &= \int_{-\infty}^{\infty} a \left[\frac{S_t^M(a, m)}{E[S_t^M(A, m)]} - \frac{S_t^F(a, f)}{E[S_t^F(A, f)]} \right] g(a) da. \quad \square
 \end{aligned}$$

PROOF OF LEMMA 2. Let $\underline{m} \leq m_t \leq \bar{m}$ for all $t = 1, 2, \dots$. Since the distributions G and Φ have full support, the probability that any male survives at any date t , conditional on

¹¹In a model where the signal for promotion depends on effort, Bardsley and Sherstyuk (2006) show that differences in the cost of effort can also lead to different promotion rates.

having survived to $t - 1$, is strictly less than one. That is, $1 - \Phi(m_t | a) < 1$ for every t and every $a \in \mathbb{R}$.

(i) Let $\varepsilon > 0$. Let $\tilde{a} > 0$ satisfy $0 < 1 - G(\tilde{a}) < \varepsilon/2$ and let $\tilde{t}$ satisfy $(1 - \Phi(m | a))^{\tilde{t}} < \varepsilon/2$ for all $a \leq \tilde{a}$. Then for $t \geq \tilde{t}$,

$$\begin{aligned} & \int_{-\infty}^{\infty} g(a) S_t^M(a, m) da \\ &= \int_{-\infty}^{\tilde{a}} g(a) \Pi_{d=1}^t (1 - \Phi(m_d | a)) da + \int_{\tilde{a}}^{\infty} g(a) \Pi_{d=1}^t (1 - \Phi(m_d | a)) da \\ &\leq \int_{-\infty}^{\tilde{a}} g(a) (1 - \Phi(m | a))^t da + \int_{\tilde{a}}^{\infty} g(a) \Pi_{d=1}^t (1 - \Phi(m_d | a)) da \\ &< G(\tilde{a})\varepsilon/2 + (1 - G(\tilde{a})) < \varepsilon. \end{aligned}$$

(ii) Given $x \in \mathbb{R}$, the probability that a random male survivor has ability greater than x is

$$\begin{aligned} & \frac{\int_x^{\infty} g(a) S_t^M(a, m) da}{\int_x^{\infty} g(a) S_t^M(a, m) da + \int_{-\infty}^x g(a) S_t^M(a, m) da} \\ &> \frac{\int_x^{\infty} g(a) S_t^M(a, m) da}{\int_x^{\infty} g(a) S_t^M(a, m) da + S_t^M(x, m) \int_{-\infty}^x g(a) da} \\ &= \frac{\int_x^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(x, m)} da}{\int_x^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(x, m)} da + G(x)}. \end{aligned} \quad (15)$$

Using the boundedness of m and the fact that $\Phi(m_d | a)$ decreases with a due to the monotone likelihood ratio property, as t gets large we have

$$\frac{S_t^M(a, m)}{S_t^M(x, m)} = \prod_{d=1}^t \frac{[1 - \Phi(m_d | a)]}{[1 - \Phi(m_d | x)]} \rightarrow \infty \text{ for } a > x. \quad (16)$$

Thus, (15) converges to one as t becomes large. The result follows.

(iii) For given t , write the expected ability of surviving males, $E^M[A | m, t]$, in two parts. The first term in (17) is the expected ability of survivors who satisfy $|A| > 4x$, times the probability of that event, and the second part is the expected ability of survivors who satisfy $|A| \leq 4x$, times the probability of that event:

$$E^M[A | m, t, |A| > 4x] \times \Pr[|A| > 4x] + E^M[A | m, t, |A| \leq 4x] \times \Pr[|A| \leq 4x], \quad (17)$$

or equivalently,

$$\begin{aligned}
 E^M[A | m, t] &= \frac{\int_{4x}^{\infty} a g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} a g(a) S_t^M(a, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} \\
 &\quad + \frac{\int_{-4x}^{4x} a g(a) S_t^M(a, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} \\
 &= \frac{\int_{4x}^{\infty} a g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} a g(a) S_t^M(a, m) da}{\int_{4x}^{\infty} g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} g(a) S_t^M(a, m) da} \\
 &\quad \times \frac{\int_{4x}^{\infty} g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} g(a) S_t^M(a, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} \\
 &\quad + \frac{\int_{-4x}^{4x} a g(a) S_t^M(a, m) da}{\int_{-4x}^{4x} g(a) S_t^M(a, m) da} \times \frac{\int_{-4x}^{4x} g(a) S_t^M(a, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da}.
 \end{aligned}$$

Since $\Pr[|A| > 4x] + \Pr[|A| \leq 4x] = 1$, and $E^M[A | m, t, |A| \leq 4x] > -4x$, it is enough to show that as t becomes large, $E^M[A | m, t, |A| > 4x]$ becomes large (larger than $2x$) and $\Pr[|A| \leq 4x]$ becomes small (smaller than $1/6$). Then $E^M[A | m, t] > (2x)(5/6) - (4x)(1/6) = x$.

I first show that $E^M[A | m, t, |A| > 4x] > 2x$ for large t :

$$\begin{aligned}
 E^M[A | m, t, |A| > 4x] &= \frac{\int_{4x}^{\infty} a g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} a g(a) S_t^M(a, m) da}{\int_{4x}^{\infty} g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} g(a) S_t^M(a, m) da} \\
 &> \frac{4x \int_{4x}^{\infty} g(a) S_t^M(a, m) da + \int_{-\infty}^{-4x} a g(a) S_t^M(a, m) da}{\int_{4x}^{\infty} g(a) S_t^M(a, m) da + S_t^M(-4x, m) G(-4x)} \\
 &= \left[4x + \frac{\int_{-\infty}^{-4x} a g(a) \frac{S_t^M(a, m)}{S_t^M(-4x, m)} da}{\int_{4x}^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(-4x, m)} da} \right] \left/ \left[1 + \frac{G(-4x)}{\int_{4x}^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(-4x, m)} da} \right] \right. \quad (18)
 \end{aligned}$$

Using the boundedness of m and the fact that $\Phi(m_d | a)$ decreases with a due to the monotone likelihood ratio property, as t gets large we have

$$\frac{S_t^M(a, m)}{S_t^M(-4x, m)} = \prod_{d=1}^t \frac{[1 - \Phi(m_d | a)]}{[1 - \Phi(m_d | -4x)]} \rightarrow 0 \text{ for } a < -4x \quad (19)$$

$$\frac{S_t^M(a, m)}{S_t^M(-4x, m)} = \prod_{d=1}^t \frac{[1 - \Phi(m_d | a)]}{[1 - \Phi(m_d | -4x)]} \rightarrow \infty \text{ for } a > 4x. \quad (20)$$

The value of (18) is less than $4x$, since the numerator adds a negative term to $4x$, and the denominator of (18) is greater than one. However, using the fact that $\int_{-\infty}^{-4x} a g(a) da$

is finite¹² and using (19) and (20), it follows that the negative term in the numerator vanishes for large enough t . Using (20), the second term of the denominator of (18) vanishes as t becomes large, and we can therefore assert that $E^M[A \mid m, t, |A| > 4x] > 2x$ for large enough t .

I now show that $\Pr[|A| \leq 4x] < 1/6$ for large enough t . We have

$$\begin{aligned} \Pr[|A| \leq 4x] &= \frac{\int_{-4x}^{4x} g(a) S_t^M(a, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} < \frac{\int_{-4x}^{4x} g(a) S_t^M(4x, m) da}{\int_{-\infty}^{\infty} g(a) S_t^M(a, m) da} \\ &= \frac{\int_{-4x}^{4x} g(a) da}{\int_{-\infty}^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(4x, m)} da} = \frac{\int_{-4x}^{4x} g(a) da}{\int_{-\infty}^{4x} g(a) \frac{S_t^M(a, m)}{S_t^M(4x, m)} da + \int_{4x}^{\infty} g(a) \frac{S_t^M(a, m)}{S_t^M(4x, m)} da}. \end{aligned}$$

The result follows from (16), substituting $4x$ for x , since the denominator becomes large.

(iv) I omit the proof, which is essentially the same as for part (iii), replacing $E^M[A \mid m, t]$ with $e^M[A \mid m, t]$, and using the fact that ϕ is bounded. $\square$

PROOF OF LEMMA 3. The sequence $c = c_1, c_2, \dots, c_t, \dots$ contains at most a finite number of values larger than $\bar{c} + \varepsilon$. Let $\bar{t}$ be the largest value of t in this set, that is, the latest date such that $c_t > \bar{c} + \varepsilon$. (If the sequence is nondecreasing as in Proposition 5, one can take $\varepsilon = 0$ and $\bar{t} = 0$.) Because ϕ has full support, there is a positive probability of survival to time $\bar{t}$, regardless of ability, that is, $\bar{S}_{\bar{t}}^M(a, c) > 0$ for all a .

That $\bar{S}_{\bar{t}}^M(a, c) \rightarrow 0$ for $a < \bar{c}$ follows because $\bar{Z}^t$ converges in probability to a and $a < \bar{c}$.

That $\bar{S}_{\bar{t}}^M(\bar{c}, c) \rightarrow 0$ follows from the central limit theorem. When $a = \bar{c}$, the limit distribution of $\sqrt{t}(\bar{Z}^t - \bar{c})/\nu$ is normal, centered at 0, where ν^2 is the variance of Φ . With positive probability we have $\sqrt{t}(\bar{Z}^t - \bar{c})/\nu < 0$. But since survival requires that $\bar{Z}^t - \bar{c} \geq 0$ for all $t = 1, 2, \dots$, the joint probability of survival at $t = 1, 2, \dots$ falls to zero.

I now show that for $a > \bar{c} + \varepsilon$, $\bar{S}^M(a, c) > 0$.

For arbitrary dates t_1, t_2 , it is useful to introduce the notation $\bar{Z}_{t_1}^{t_2}$ for the average of signals accrued between those dates, namely $\bar{Z}_{t_1}^{t_2} = (1/(t_2 - t_1)) \sum_{k=t_1+1}^{t_2} Z_k$. The previous notation $\bar{Z}^t$ becomes $\bar{Z}_0^t$.

Let $\mathcal{P}(a, c; \bar{t}, Z)$ be the probability that an agent with ability $A = a$ survives in the limit, conditional on having survived to $\bar{t}$ with average signal $\bar{Z}_0^{\bar{t}} = Z \in \mathbb{R}$. Survival implies that $Z \geq c_{\bar{t}}$. Given a and Z , write

$$\mathcal{P}(a, c; \bar{t}, Z) = \Pr \left[\left(\frac{\bar{t}}{t} \right) Z + \left(\frac{t - \bar{t}}{t} \right) \bar{Z}_{\bar{t}}^t \geq c_t \text{ for all } t = \bar{t} + 1, \bar{t} + 2, \dots \right], \quad (21)$$

where $\{\bar{Z}_{\bar{t}}^t : t = \bar{t} + 1, \bar{t} + 2, \dots\}$ is the random variable.

It is enough to show that $\mathcal{P}(a, c; \bar{t}, Z) > 0$ for all $a > \bar{c} + \varepsilon$ and all $Z \geq c_{\bar{t}}$. Since ϕ has full support, $\bar{S}_{\bar{t}}^M(a, c) > 0$ for all abilities a . Therefore if $\mathcal{P}(a, c; \bar{t}, Z) > 0$ for all $a > \bar{c} + \varepsilon$ and $Z \geq c_{\bar{t}}$, we have $\bar{S}^M(a, c) > 0$ for $a > \bar{c} + \varepsilon$.

¹²Notice that $\int_{-\infty}^{\infty} a^2 g(a) da$ is finite because variance is finite.

The first line below follows from (21), because $c_{\bar{t}} > c_t$, hence, using $Z \geq c_{\bar{t}} > c_t$, we have $Z \geq c_t$ for all $t > \bar{t}$. Therefore the event $[\bar{Z}_{\bar{t}}^t \geq c_t \text{ for all } t = \bar{t} + 1, \bar{t} + 2, \dots]$ implies the event $[(\bar{t}/t)Z + ((t - \bar{t})/t)\bar{Z}_{\bar{t}}^t \geq c_t \text{ for all } t = \bar{t} + 1, \bar{t} + 2, \dots]$, but not vice versa.

The second line is a change of notation. In the third line, the constant value a is subtracted from both sides of the inequality. The fourth line uses the fact that $c_{\tau+\bar{t}} \leq \bar{c} + \varepsilon$ for all $\tau > 0$. The fifth line follows because the distribution of signals is atomless.

$$\begin{aligned} \mathcal{P}(a, c; \bar{t}, Z) &> \Pr[\bar{Z}_{\bar{t}}^t \geq c_t \text{ for all } t = \bar{t} + 1, \bar{t} + 2, \dots] \\ &= \Pr[\bar{Z}_0^\tau \geq c_{\tau+\bar{t}} \text{ for all } \tau = 1, 2, \dots] \\ &= \Pr[\bar{Z}_0^\tau - a \geq c_{\tau+\bar{t}} - a \text{ for all } \tau = 1, 2, \dots] \\ &\geq \Pr[\bar{Z}_0^\tau - a \geq \bar{c} + \varepsilon - a \text{ for all } \tau = 1, 2, \dots] \\ &= \Pr[\bar{Z}_0^\tau - a > \bar{c} + \varepsilon - a \text{ for all } \tau = 1, 2, \dots]. \end{aligned}$$

For $a > c + \varepsilon$, choose $b_1, b_2 > 0$ so that $-b_1 v^2 - b_2 = \bar{c} + \varepsilon - a$. Then

$$\begin{aligned} \mathcal{P}(a, c; \bar{t}, Z) &> \Pr[\bar{Z}_0^\tau - a > \bar{c} + \varepsilon - a \text{ for all } \tau = 1, 2, \dots] \\ &= \Pr[\bar{Z}_0^\tau - a > -b_1 v^2 - b_2 \text{ for all } \tau = 1, 2, \dots] \\ &\geq \Pr[\bar{Z}_0^\tau - a > -b_1 v^2 - b_2/\tau \text{ for all } \tau = 1, 2, \dots] \end{aligned}$$

Using Lemma (6) of Dubins and Freedman (1965, p. 801),

$$\Pr[\bar{Z}_0^\tau - a \leq -b_1 v^2 - b_2/\tau \text{ for some } \tau = 1, 2, \dots] \leq \frac{1}{1 + b_1 b_2}.$$

Thus,

$$\mathcal{P}(a, c; \bar{t}, Z) > \Pr[\bar{Z}_0^\tau - a > -b_1 v^2 - b_2/\tau \text{ for all } \tau = 1, 2, \dots] \geq 1 - \frac{1}{1 + b_1 b_2} > 0 \quad \square$$

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