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Credible ratings

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This paper considers a model of a rating agency with multiple clients, in which each client has a separate market that forms a belief about the quality of the client after the agency issues a rating. When the clients are rated separately (individual rating), the credibility of a good rating in an inflationary equilibrium of the signaling game is limited by the incentive of the agency to exaggerate the quality of the client. With a centralized rating, the agency rates all clients together and shares the rating information among all markets. This allows the agency to coordinate the ratings and achieve a higher average level of credibility for its good ratings than with individual rating. Under decentralized rating, the ratings are again shared among all markets, but each client is rated by a self-interested rater of the agency with no access to the quality information of other clients. When the underlying qualities of the clients are correlated, decentralized rating leads to a smaller degree of rating inflation and hence a greater level of credibility than under individual rating. Comparing centralized rating with decentralized rating, we find that centralized rating dominates decentralized rating for the agency when the underlying qualities are weakly correlated, but the reverse holds when the qualities are strongly correlated.

Keywords. Signaling, credibility, individual rating, centralized rating, decentralized rating.

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1. Introduction

Consider a rating agency that issues a report on each of its clients. The rating agency is informed of the quality of each client and its report on the client is received as a signal

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by the market that the client faces. The agency cares about the payoff to each client. Examples of a rating agency with multiple clients include an economics department that evaluates its PhD graduates, a stock brokerage firm that deals with multiple stocks, and a consumer electronics magazine that issues ratings on multiple products. We are interested in an environment in which the payoff to each client depends only on the perceived quality of that client, and not on the perceived qualities of other clients, so that there is no direct payoff link among the clients. The only possible link is informational: when the markets are given access to all client ratings, the perceived quality of each client can depend on the ratings of other clients, either exogenously through some statistical correlation among client qualities, or endogenously through the reporting strategy of the agency, or both. In the economics department example, the payoff link is likely to be absent if the PhD graduates are in different fields so that their markets are separate, or if the markets are sufficiently thick that each graduate receives a competitive wage, while the informational link will be present if there are strong cohort effects in the graduate program or if the department ranks the students by comparing them. Similarly, for the stock brokerage firm example and the consumer magazine example, there may be little demand substitutability or complementarity in the aggregate so that the price of a rated stock or an electronic product depends only on the valuation of that stock or product,[1] but a positive correlation among the client qualities can still arise, for example, if the future returns of all the stocks are affected by an economy-wide shock or the electronic products share significant common parts or designs. In our model, because the agency cares about the perceived qualities of its clients, the credibility of the ratings is at issue. The objective of this paper is to compare the credibility of ratings under three schemes that differ in whether the markets have access to all the reports and in whether the raters in the agency share the knowledge about client qualities.

Under "individual rating," the market for each client does not observe the ratings for other clients. This is a natural benchmark due to the absence of any direct payoff linkage. The rating scheme can be analyzed as a simple signaling model with one sender (the rating agency with a single client) and a receiver (the market for the client), with the market interested only in making the right inference about the client's underlying quality. We make assumptions on the payoff function of the agency regarding its reputational concerns and how these concerns interact with the derived benefits from an improved perception of the client quality. These assumptions imply that the incentive to exaggerate the quality always outweighs the reverse incentive to downplay it. This "single crossing" property allows us to focus on the "inflationary equilibrium," which is a semi-pooling equilibrium where the client's quality is truthfully revealed whenever it

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[1]The literature on asset pricing focuses on the case where the price of a stock depends on the probability distribution of the future cash flow and some "pricing kernel." In a large market, the cash flow on any single stock does not affect the pricing kernel, and so the payoffs for different stocks are separable. For the electronics example, payoff separability is a more appropriate assumption if the products belong to different categories, or if consumers have strong brand loyalty.

is good and sometimes exaggerated when it is bad. The benchmark model of individual rating can be interpreted as a model of credibility. The equilibrium perception of a good rating measures credibility and there is a one-to-one correspondence between credibility and the equilibrium ex ante payoff of the agency. The inability of the rating agency to commit to an honest rating policy dilutes the meaning of a good rating without changing the meaning of a bad rating, and therefore reduces the rating agency's ex ante payoff. We ask the following question in the rest of the paper: can the rating agency obtain a higher ex ante payoff than in the inflationary equilibrium under individual rating by improving the credibility of good ratings?

Under "centralized rating," the agency rates all clients together and shares the reports among all markets. Each market can use the ratings of other clients as well as its own client to make an inference about the quality of the latter. When the rating information is shared among all markets, the agency can effectively coordinate the ratings of its client. For example, the agency can employ a correlated randomization strategy between good and bad ratings across clients of bad quality, even when client qualities are statistically independent. It turns out that correlated randomization is necessary to improve the agency's payoff beyond the benchmark inflationary equilibrium under individual rating. We show that there exists an equilibrium that weakly dominates the benchmark inflationary equilibrium for the agency, and that an equilibrium that strictly dominates it exists when the prior probability that no client has good quality is small.

Under "decentralized rating," the ratings are shared among all markets, as under centralized rating, but each client is rated by a self-interested rater of the agency with no access to the quality information of other clients. This means that only independent randomization across clients of bad quality is possible, as under individual rating. However, unlike under individual rating, ratings information is shared among all markets, thus the perception of a good rating depends on the total number of good ratings in all markets. This endogenous payoff link among the clients makes it more difficult for each rater to fool the market with an exaggerated rating. As a result, the equilibrium probability of an inflationary rating can be lower and the average credibility of a good rating can be higher than in the benchmark inflationary equilibrium under individual rating, leading to a greater equilibrium payoff for the agency.

The comparison between centralized rating and decentralized rating in terms of equilibrium credibility of good ratings and ex ante payoff to the agency depends on the degree of correlation. When the underlying qualities are independently distributed, any inflationary equilibrium under decentralized rating is payoff-equivalent to the benchmark inflationary equilibrium under individual rating, as the ratings of other clients cannot discipline each individual rater and thus there is no gain in credibility. In contrast, with independent qualities, an equilibrium that strictly dominates the benchmark equilibrium typically exists under centralized rating. With correlation across the underlying qualities, there is less room to manipulate ratings under both centralized rating and decentralized rating. When the underlying qualities are almost perfectly correlated, under centralized rating there is no inflationary equilibrium that strictly dominates the benchmark equilibrium under individual rating, as the strong correlation across client qualities severely reduces the credibility of coordinated rating. In contrast, under decentralized rating the discipline on credibility imposed by strong correlation allows the construction of an inflationary equilibrium that is arbitrarily close to truth-telling. Thus, centralized rating is dominated by decentralized rating for the agency with strong correlation.

Our comparison results regarding individual rating, centralized rating, and decentralized rating have strong implications for how an agency can gain credibility for its ratings and improve its welfare. Since there exist inflationary equilibria that weakly dominate the benchmark under either centralized or decentralized rating schemes, it is always to the advantage of the agency to share ratings information among all the markets it serves. Whether the agency should share information about client qualities among its raters, or commit to a policy that restricts information access and preserves the raters' independent concerns for career reputation, depends on the underlying correlation structure across client qualities. Our results suggest that the agency should group together clients with weakly correlated qualities and centralize their rating, but for clients with strongly correlated qualities the agency should decentralize their rating among the raters.

It is interesting to interpret our comparison results between centralized rating and decentralized rating in terms of different market structures for rating agencies as opposed to different information structures for a single rating agency. The centralized rating scheme naturally corresponds to the monopoly market structure, while the decentralized scheme can be equivalently viewed as the competitive market structure. Although under the decentralized scheme there is no direct competition among the agencies because the clients have separate markets, the agencies indirectly compete for credibility as the ratings are observed by all markets. Our results then suggest that the comparison between the two market structures depends on the degree of correlation across the underlying states of nature. The monopoly structure performs better due to an economy of scale when the states are weakly correlated. When the states are strongly correlated, the competitive structure does better because competing ratings constrain the incentive to inflate and improve the credibility of good ratings.

The paper is organized as follows. Section 2 presents the basic ingredients of our model of rating agencies. We introduce the out-of-equilibrium belief refinement used throughout the paper, and characterize an inflationary equilibrium under individual rating that serves as the benchmark of comparison. In Section 3 we deal with centralized rating. This turns out to be a signaling model with one-dimensional private information and multi-dimensional signals. We establish the existence of an inflationary equilibrium that weakly dominates the benchmark inflationary equilibrium of individual rating for the agency in terms of expected payoff. We provide a sufficient condition for the existence of an equilibrium that strictly dominates the benchmark inflationary equilibrium. Section 4 presents the model of decentralized rating. We introduce a correlation structure that accommodates possibilities of both positive and negative correlation across client qualities in a multi-dimensional setting. With this structure, and under further assumptions on the payoff functions of the agency, we show that there exists an inflationary equilibrium that weakly dominates the benchmark inflationary equilibrium of individual rating for the agency in terms of expected payoff. In Section 5 we study how the comparison between centralized rating and decentralized rating depends on the correlation across client qualities. Using a specific correlation structure and focusing on the limit case when the number of clients is arbitrarily large, we show that increasing correlation leads to more ratings inflation, lower credibility of good ratings, and lower equilibrium payoff to the agency under centralized rating, while the opposite occurs under decentralized rating. Section 6 provides some remarks on related literature. Proofs of all lemmas can be found in the Appendix.

2. A model of rating agencies

A rating agency deals with N clients. In our model the N sets of relationships between each client i, $i=1,\dots,N$, and the corresponding market (end-user of the rating for the client) are identical. The underlying quality S_i of each client i is either good (G) or bad (B); the rating s_i for the client is either good (g) or bad (b). The objective function of market i is to minimize the expectation of the squared difference between a real-valued decision variable δ_i and a random variable that is equal to 1 if $S_i=G$ and 0 if $S_i=B$. Let q_i denote the market's (endogenous) belief that the quality of the client is good. The optimal decision for market i is to set δ_i equal to q_i. The realized loss is $(1-q_i)^2$ if $S_i=G$, and q_i^2 if $S_i=q_i^2$. We write the rating agency's payoff function from client i as $U(S_i,s_i,q_i)$ for $S_i=G,B$ and $s_i=g,b$.[2] The total payoff to the agency is the sum $\sum_{i=1}^N U(S_i,s_i,q_i)$.

For the statistical distribution of client qualities, at this point we assume only that the client qualities are exchangeable random variables: the probability of any realization of the random vector $(S_1,\dots,S_N)$ depends only on the number of clients of good quality. The joint probability distribution of $(S_1,\dots,S_N)$ can then be represented by a vector $(\pi_0,\dots,\pi_N)$, where π_n is the probability that there are exactly n clients of good quality. We assume that $\pi_n>0$ for each $n=0,1,\dots,N$. Define π as the probability that any given client is of good quality, which satisfies

$$\pi=\frac{1}{N}\sum_{n=1}^{N} n\pi_n. \tag{1}$$

The assumption of exchangeability introduces symmetry across clients, which simplifies our analysis without imposing statistical independence. In the applications of the model that we have in mind, correlated client qualities might be an important feature. For example, student qualities might be correlated through peer effects, stock valuations through some underlying common fundamental, and electronic products through common design features. It turns out that the specific correlation structure does not play any role in our equilibrium construction under a centralized rating scheme. We need to

[2] In our model ratings are not cheap talk. Further, we make the implicit assumption that for each client the message space is the same as the type space and we focus on the comparison between centralized and decentralized rating schemes. The analysis of games in which a richer set of costly signals is available requires additional setup and is beyond the scope of this paper.

make further assumptions on the correlation structure when we analyze decentralized rating.

A few remarks about the setup are in order. First, the specific preference function adopted here for the markets is meant to capture the idea that each client faces competitive bids after the market updates its belief about the quality of the client based on the reports of the agency. This reduces the role of the receiver in our signaling model to forming rational expectations of the client quality, and allows us to focus on the signaling incentives of the agency. Second, the payoff of the agency in the relationship with client i is assumed to depend on the market's belief q_i about client i's quality, which summarizes the payoff to the client. This models the idea that the agency is not an impartial provider of information, in that it cares about the payoff to the client. Third, both the underlying quality S_i and the signal s_i enter the payoff function of the agency. This form allows for any two-state, two-signal setup. The general idea is that the payoff of the agency is affected both by the payoff to the client and by its own reputational concerns, and we are using the function U as a reduced-form representation of the agency's payoff. Later we make further assumptions on how the concern for the client's payoff and the reputational concerns interact with each other. Finally, the payoff of the agency is assumed to be additively separable in the utilities from the N sets of client relationships. This separability assumption is justified if the payoff to each client i depends only on the belief q_i about the client's quality. As mentioned in the introduction, there are environments in the labor market, the financial market, and the goods market in which this assumption is reasonably appropriate. We do not claim that it holds in all relevant situations for rating agencies. Rather, the separability assumption is made to focus exclusively on informational issues of ratings.

We need to make further assumptions on the common payoff function U. We drop the subscript i for now as there is no risk of confusion. First, we assume that the derivative of $U(S,s,q)$ with respect to q, $U_q(S,s,q)$, exists and is strictly positive for each $q \in (0,1)$.[3]

Assumption 1. $U_q(S,s,q)$ exists and is strictly positive for each $S = G, B$, $s = g, b$, and $q \in (0,1)$.

Signaling games often have a multiplicity of equilibria. One way to minimize the equilibrium selection issue is to ensure that if the agency weakly prefers g to b when the quality is B, then it strictly prefers g to b in state G, and, conversely, if the agency weakly prefers b to g in state G, then it strictly prefers b to g when the quality is B. This condition may be referred to as "single-crossing." We use it to construct equilibria that involve only one form of misrepresentation, referred to as "inflationary rating," which is issuing a good rating when the quality is bad, rather than "deflation," or issuing a bad rating when the quality is good. For the single-crossing result to be effective in constructing inflationary equilibria, we need it to hold regardless of how different ratings

[3]This rules out situations where the market's response to the agency's rating is discrete—for example, where the only choice of the market is whether or not to acquire the client's service at some fixed wage.

induce different beliefs:

$$U(G,g,q)-U(G,b,q')>U(B,g,q)-U(B,b,q') \tag{2}$$

for all $q,q'\in[0,1]$. Condition (2) can be thought of as payoff complementarity between the underlying quality S and the rating g, modified to suit the signaling model so that it holds whenever a switch of the underlying quality for the same rating does not affect the belief q while a switch of the rating for the same quality generally affects q.[4]

The following assumption on the payoff functions $U(S,s,q)$, together with Assumption 1, immediately leads to condition (2).[5]

Assumption 2. $U_q(G,g,q)>U_q(B,g,q)$, $U_q(G,b,q)<U_q(B,b,q)$ for any $q\in(0,1)$, and

$$U(G,g,0)-U(G,b,0)>U(B,g,0)-U(B,b,0). \tag{3}$$

One may interpret the difference $U(G,g,\cdot)-U(B,g,\cdot)$ as a measure of the agency's reputational concern for honesty. Given the same rating g and any belief q, $U(B,g,q)$ differs from $U(G,g,q)$ because the agency is concerned that the true quality of the client may be discovered, thus revealing a dishonest rating. Similarly, the difference $U(B,b,\cdot)-U(G,b,\cdot)$ is a measure of the agency's reputational concern for competence: for the same rating b and any q, $U(G,b,q)$ differs from $U(B,b,q)$ because when the true quality of the client is discovered, it reveals an inaccurate rating. Assumption 2 requires both differences to be increasing in the client's perceived quality q. This assumption is motivated by the idea that it is more likely (or faster) that the market learns the true quality of the client when the perceived quality is higher. For the consumer magazine example mentioned in the introduction, if an electronic product is new to the market and is of an experience good variety, a higher perceived quality leads to greater sales and faster consumer learning about its true quality. Similarly, a higher market belief about the quality of a job candidate is more likely to result in a better and more challenging job placement, which can quickly reveal the true quality of the candidate, and a higher valuation about a rated stock may lead to a greater transaction volume, which motivates more subsequent research.

Assumption 3. $U(B,g,1)>U(B,b,0)>U(B,g,0)$.

This assumption makes the individual rating scheme, analyzed in the next subsection, a benchmark model of credibility.[6] When the quality is B, the agency has an incentive

[4]Condition (2) is stronger than we need for the purpose of the analysis; single-crossing requires it to hold only when the right-hand-side is non-negative.

[5]To see this, note that since $U_q(G,g,q)>U_q(B,g,q)$, we have $U(G,g,q)-U(B,g,q)\geq U(G,g,0)-U(B,g,0)$ for any q. Similarly, since $U_q(G,b,q)<U_q(B,b,q)$, we have $U(G,b,q')-U(B,b,q')\leq U(G,b,0)-U(B,b,0)$ for any q'. Condition (2) then follows from inequality (3) in Assumption 2. Also, the inequalities are sufficient but not necessary for the single-crossing condition (2). Our analysis of individual rating and decentralized rating goes through as long as (2) holds, but the two inequality conditions on U_q are used for the equilibrium construction in the case of centralized rating.

[6]If the first inequality of the assumption is violated, the agency always prefers to report truthfully when

to issue an inflationary rating if it results in a sufficiently favorable belief, but there is no incentive to inflate if beliefs cannot be favorably manipulated. Together with Assumption 1, it implies that there is a unique $q^* \in (0,1)$ that satisfies

$$U(B,g,q^*) = U(B,b,0). \tag{4}$$

The value of q^* can be thought of as an inverse measure of the incentive to inflate when the client quality is bad.

The final assumption is a strengthening of the single-crossing condition (2). It turns out that under decentralized rating, the single-crossing condition is sufficient for the existence of an inflationary equilibrium, just as under individual rating. However, under centralized rating, the condition needs to be strengthened to ensure the construction of inflationary equilibria.

Assumption 4. For any $q \in (q^*,1)$,

$$\frac{U(G,g,q)-U(G,b,0)}{U(B,g,q)-U(B,b,0)} > \frac{U_q(G,g,q)}{U_q(B,g,q)}.$$

By Assumption 3, the denominator of the left-hand-side of the this inequality is positive. It then follows from Assumption 2 that the numerator also is positive, and in fact, both the left-hand-side and the right-hand-side are greater than 1. Assumption 4 strengthens condition (2) for market beliefs that are more favorable than the benchmark belief defined by equation (4). Alternatively, the assumption can be viewed as imposing an upper bound on $U(G,b,0)$, which is the payoff to the agency from a client of quality G when it issues the rating b. Assumption 4 thus requires the payoff to be sufficiently low, or the reputational concerns for competence to be sufficiently great. This assumption is used in the construction of inflationary equilibria under centralized rating to regulate the incentives to issue deflationary ratings.

Specific examples where all four assumptions are satisfied can easily be constructed. For example, suppose that $U(S,s,q)$ is linear in q for each $S=G,B$ and $s=g,b$. Then, as long as misreporting entails some cost for the agency and the cost is not so large that truthtelling is a dominant strategy (that is, $U(G,g,q) > U(G,b,q)$ and $U(B,b,q) > U(B,g,q)$ for all $q \in [0,1]$ but $U(B,g,1) > U(B,b,0)$), then all assumptions are satisfied when the slope restrictions in Assumption 2 are respected. Alternatively, consider a simple model where the agency's payoff from truthtelling depends only on the market belief q, and its payoff from misreporting is a fraction of the truthtelling payoff because with a positive probability the misreporting is discovered and the agency's payoff is zero. That is, there exists a strictly increasing and positive-valued function $\tau(q)$ and a number κ between 0 and 1, such that $U(G,g,q) = U(B,b,q) = \tau(q)$, and

the client quality is bad. If the second inequality is violated, then regardless of the prior belief about the client quality, under individual rating it is an equilibrium to issue a good rating whether the client quality is good or bad, and this is the only inflationary equilibrium. Neither case provides an interesting benchmark model of credibility. Note that the assumption below does not rule out a pooling inflationary equilibrium.

$U(G,b,q) = U(B,g,q) = (1-\kappa)\tau(q)$. Then, Assumptions 1, 2, 4, and the second inequality of Assumption 3 are immediately satisfied. As long as κ is not so large that the agency always prefers to report truthfully, then also the first part of Assumption 3 is satisfied.[7]

2.1 *Individual rating: A model of credibility*

Under individual rating, the market for each client has no access to ratings for other clients. Since the clients are exchangeable, the model reduces to N identical signaling games involving the agency and the market. In each such game, an inflationary rating strategy is such that the agency issues g under quality G and randomizes between g and b under quality B. Suppose that there exists $p \in (0,1)$ such that

$$\frac{\pi}{\pi + (1-\pi)p} = q^*, \tag{5}$$

where π is given in equation (1). Then, we have a semi-separating equilibrium in which the agency issues g under B with probability p: by equation (4) the agency is indifferent between g and b under quality B, which by the single-crossing condition (2) implies that the agency strictly prefers g to b under quality G. We refer to this type of inflationary equilibrium as a "full support inflationary equilibrium," as the support of the equilibrium strategy is the same as the space of the signals. Since equation (5) can be satisfied by some $p \in (0,1)$ only if $\pi < q^*$, a full support equilibrium does not exist if $\pi \geq q^*$. Instead, we can construct a "non-full support equilibrium," in which the agency issues g with probability 1 under B. This is accomplished by specifying the out-of-equilibrium belief that the quality of the client is B with probability 1 when b is observed. Since the equilibrium belief that the quality is G when g is observed is equal to the prior probability π, the agency weakly prefers g to b under quality B, which implies that it strictly prefers g to b under G by (2). Further, due to the same single-crossing condition (2), the above specification of the out-of-equilibrium belief is the only one consistent with the refinement concept of "Divinity" (Banks and Sobel 1987).[8] We use this refinement throughout the paper, and we refer to a sequential equilibrium that passes the refinement test simply as an equilibrium. It follows that there is a unique inflationary equilibrium under individual rating, which is full support if $q^* > \pi$ and non-full support if $q^* \leq \pi$.[9]

The model of individual rating can be interpreted as a model of credibility. Upon observing a good rating, the market's perception of the client's quality is q^* in a full support equilibrium, and π in a non-full support equilibrium. This market belief quantifies equilibrium credibility in our model. From the equilibrium indifference condition

[7]The example can be modified to make the agency's payoff depend on the client's underlying quality, and to allow a different probability of discovering misreporting when the agency is inflating or deflating.

[8]More precisely, for any out-of-equilibrium belief $\hat{q}$ that the quality is G after b is observed, $U(G,b,\hat{q}) \geq U(G,g,\pi)$ implies that $U(B,b,\hat{q}) > U(B,g,\pi)$. Thus, $\hat{q} = 0$ under the refinement of Banks and Sobel (1987).

[9]With additional assumptions, we can show that no other equilibrium exists under individual rating. In particular, if $U(G,g,1) > U(G,b,1)$, then we can rule out all "deflationary" equilibria in which the agency issues b with a positive probability under quality G. However, since the focus of this paper is on the credibility of good ratings, we are interested only in constructing inflationary equilibria under different rating schemes.

(4), we see that the value of q^* depends only on the function $U(B,g,\cdot)$ and the value of $U(B,b,0)$. When the prior probability of good quality is higher than q^*, an increase in the prior translates into an increase in the equilibrium credibility of good ratings by the same amount, which allows the agency to simply pass any client of bad quality as one of good quality. In contrast, when the prior probability is lower than q^*, an increase in the prior has no effect on the equilibrium credibility. The increase in the prior probability means that a good rating is too attractive if the agency keeps the probability of reporting g in state B unchanged, and so the probability of an inflated good rating must increase to restore the equilibrium indifference condition. As a result, the equilibrium credibility, and hence the payoff to the agency, is pinned down by the indifference condition so long as the agency reports b with positive probability in equilibrium. In equilibrium the agency gets its complete information payoff $U(B,b,0)$ under quality B, but its equilibrium payoff under quality G is $U(G,g,q)$, which is strictly lower than the complete information payoff $U(G,g,1)$. Thus, the ex ante payoff to the agency (before the client's quality is revealed) is lower than what the agency would obtain if it could commit to truthful revelation of the quality.

Our definition of credibility corresponds one-to-one with the expected marginal value of information provided by the rating in equilibrium. In the absence of any rating, the optimal decision of each market i is to set δ_i to π. The expected loss is therefore $\pi(1-\pi)$. In a full support inflationary equilibrium, when the client quality is good the realized loss of the market is $(1-q^*)^2$; when the quality is bad the realized loss is $(q^*)^2$ if g is issued, and 0 if b is issued. Using equation (5), the equilibrium expected loss is $\pi(1-q^*)$. Thus, a greater value for q^* means a lower expected loss to the market, and a greater marginal value of the information provided by the rating. Of course, in a non-full support equilibrium, there is no information in the equilibrium rating.

3. Centralized rating: A model of multi-dimensional signals

This section considers centralized rating, in which a single rater of the agency rates all N clients and shares the rating information among all markets. Although the payoff to each client depends only on the market's perception of the quality of this client, under centralized rating all the reports are used to make an inference about the quality of each client. This means that the agency can potentially coordinate the N ratings in an attempt to influence market perception.

It may not be intuitive that centralized rating creates opportunities for the agency to increase the credibility of good ratings relative to individual rating, especially if the client qualities are statistically independent. Indeed, it is easy to see that in the case of independent qualities, the equilibrium outcome of individual rating can be supported under centralized rating if the agency independently randomizes between g and b for each client of bad quality with the same probability of choosing b as under individual rating. In this case, the market belief about the quality of any client i with a good rating remains q^*, regardless of the other ratings, as they provide no information about client i's quality under independent qualities and independent randomization. Moreover, this is the only equilibrium outcome under independent randomization. Indeed, a more general result is established below: even when the qualities are correlated and randomizations are coordinated among the clients, any inflationary equilibrium is payoff-equivalent to the benchmark inflationary equilibrium with belief q^* as long as N bad ratings are issued with a positive probability in equilibrium. The key to improved credibility under centralized rating relative to individual rating is to construct an inflationary equilibrium in which the agency never reports N bad ratings, and we provide a characterization of the structure of any such equilibrium. The main result of this section establishes a necessary and sufficient condition for the existence of an equilibrium with improved credibility. This condition requires the prior probability of having N bad qualities to be sufficiently low, so that it is credible for the agency never to issue N bad ratings.

Formally, for the rating agency, the state is now an N-dimensional vector $(S_1,\dots,S_N)$ where $S_i \in \{G,B\}$ for $i=1,\dots,N$. The signal is similarly an N-dimensional vector $(s_1,\dots,s_N)$ where $s_i \in \{g,b\}$ for $i=1,\dots,N$. Given that $S_1,\dots,S_N$ are exchangeable, we impose a symmetry requirement that the market belief about any client i's quality depends only on the rating s_i of the client and the total number good ratings issued by the agency. For any $i=1,\dots,N$, let $q(m)$ be the market belief that $S_i=G$ when $s_i=g$ and $\#\{j: s_j=g\}=m$. Similarly, define $\hat{q}(m)$ to be the market belief that $S_i=G$ when $s_i=b$ and $\#\{j: s_j=g\}=m$. Given the state, the agency chooses the signal vector $(s_1,\dots,s_N)$ to maximize the sum of utilities $\sum_{i=1}^N U(S_i,s_i,q_i)$ where $q_i=q(m)$ if $s_i=g$ and $q_i=\hat{q}(m)$ if $s_i=b$ for all $m=\#\{j: s_j=g\}$. It directly follows from the single-crossing condition (2) that while the agency may have an incentive to mislead the markets about the total number of clients of good quality, it has no incentive to mislead the markets about the identity of clients of good quality. That is, for any $i=1,\dots,N$, when $\#\{j: S_j=G\} \le \#\{j: s_j=g\}$, then $S_i=G$ implies $s_i=g$.[10] The same is true about the identity of clients of bad quality when the agency deflates the number of clients of good quality. As a result, we can reduce the state space to a one-dimensional variable representing the number of clients of good quality. Denote the signaling strategy of the agency as $p(m;n)$, the probability of giving m good ratings when n clients are of good quality. Note that the strategy is multi-dimensional because for each number n we need to specify a vector of probability numbers $p(m;n)$ for $m=0,\dots,N$. Obviously, we require $\sum_{m=0}^N p(m;n)=1$ for all $n=0,\dots,N$.

Let $W(m;n)$ be the expected payoff to the agency when it chooses m good ratings and the number of good quality clients is n. For $m \ge n$, we have

$$W(m;n)=nU(G,g,q(m))+(m-n)U(B,g,q(m))+(N-m)U(B,b,\hat{q}(m)).$$

For $m \le n$, we have

$$W(m;n)=mU(G,g,q(m))+(n-m)U(G,b,\hat{q}(m))+(N-n)U(B,b,\hat{q}(m)).$$

[10] To see this, let $\#\{j: S_j=G\}=n$ and $\#\{j: s_j=g\}=m$. If $\#\{j: S_j=G \text{ and } s_j=g\}=n$, the expected payoff to the agency is $nU(G,g,q(m))+(m-n)U(B,g,q(m))+(N-m)U(B,b,\hat{q}(m))$. If instead $\#\{j: S_j=G \text{ and } s_j=g\}=n'<n$, the expected payoff to the agency is reduced by $(n-n')$ times $[U(G,g,q(m))-U(G,b,\hat{q}(m))]-[U(B,g,q(m))-U(B,b,\hat{q}(m))]$, which is positive by condition (2).

An inflationary strategy satisfies $p(m;n)=0$ for all n and all $m<n$. Using Assumption 2, we have the following restriction on equilibrium strategies.[11]

Lemma 1. *For any $n<n'\leq m,m'$ and $q(m')>q(m)$, if $W(m';n)\geq W(m;n)$, then $W(m';n')>W(m;n')$.*

Thus, in any inflationary equilibrium, the incentive to inflate to a signal with a more favorable belief about good ratings is stronger when there are more clients of good quality. Note that the relative incentive to inflate depends on how favorably the signal is received, and not directly on how many good ratings the signal contains. Given an inflationary equilibrium let $T=\{m:\sum_{n=0}^{N}p(m;n)>0\}$ be the set of all signals that are issued with positive probability, and let $l=\min T$ be the smallest signal (with the lowest number of good ratings). Define $T_n=\{m:p(m;n)>0\}$ as the set of signals sent with positive probabilities when there are n clients of good quality. In an inflationary equilibrium, for each $m\in T$, the market beliefs upon observing m good ratings are

$$q(m)=\frac{\sum_{n=0}^{N}\pi_n p(m;n)n}{m\sum_{n=0}^{N}\pi_n p(m;n)}$$

and $\hat{q}(m)=0$.

The following lemma distinguishes two types of inflationary equilibria, one that is payoff-equivalent to the full support inflationary equilibrium under individual rating with $q(m)=q^*$ for each equilibrium message $m\in T$, and the other payoff-superior.

Lemma 2. *In any inflationary equilibrium, (i) if $l=0$, then $q(m)=q^*$ for all $m>0$ and $m\in T$, and (ii) if $l>0$, then either $q(m)=q^*$ for all $m\in T$ or $q(m)>q(m')>q^*$ for $m,m'\in T$ and $m<m'$.*

An inflationary equilibrium with $l=0$ does not have full support if $T\neq\{0,1,\ldots,N\}$. However, part (i) of Lemma 2 establishes that any inflationary equilibrium with $l=0$ is payoff-equivalent to the full support inflationary equilibrium under individual rating. Although each market can use the ratings of other clients as well as that of its own client to make inferences about the quality of the latter, the rating agency gains no credibility relative to individual rating. In any such equilibrium, when all clients have bad quality, the agency is indifferent between issuing zero good ratings and issuing any number of good ratings in T. These indifference conditions reduce centralized rating to individual rating in terms of payoff to the agency.[12] Part (ii) of Lemma 2 establishes that in an

[11] In the Appendix, the same lemma also establishes that for any $m\leq n<n'\leq m'$, if $W(m';n)\geq W(m;n)$ then $W(m';n')>W(m;n')$, and for any $m,m'\leq n'<n$ and $\hat{q}(m')>\hat{q}(m)$, if $W(m';n)\geq W(m;n)$, then $W(m';n')>W(m;n')$. These two additional parts are needed to restrict out-of-equilibrium beliefs in inflationary equilibria.

[12] The proof of this result (in the Appendix) is more complicated than indicated by this reasoning, because we have to allow for non-full support strategies. This requires the use of the refinement. Later, we will show that all inflationary equilibria have the threshold property that $T=\{l,\ldots,N\}$. However, if we restrict to strategies that satisfy this property, then Lemma 2 and parts (i) through (iii) of Lemma 3 below can be established using the equilibrium conditions, without resorting to the refinement.

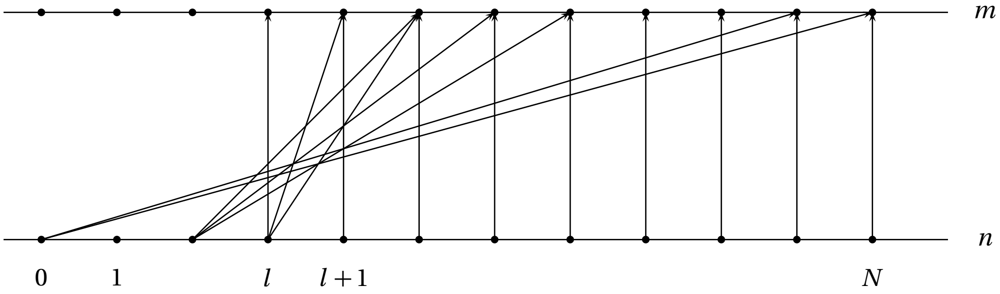

FIGURE 1. Structure of equilibrium strategy

equilibrium with $l > 0$, either the same indifference conditions are again at work and the market belief corresponding to a good rating is the same regardless of the number of good ratings issued and equal to q^*, or the market beliefs are all strictly greater than q^*. In the second case, the beliefs decrease in the number of good ratings issued, for otherwise the agency would inflate as much as possible. The second type of inflationary equilibria is more interesting, because the agency's ex ante payoff is higher than in the benchmark full support individual rating case.[13] From now on, we distinguish equilibria according to whether they are payoff-equivalent to the full support equilibrium under individual rating: equilibria with $l > 0$ and $q(l) > q^*$ are referred to as non-full support equilibria, and those with $q(m) = q^*$ for all $m \in T$ are referred to as full support equilibria regardless of whether $l = 0$ or $l > 0$. The next lemma provides a partial characterization of the structure of the equilibrium signaling strategy in a non-full support equilibrium.

LEMMA 3. *In any non-full support equilibrium, (i)* $T_l \ni l$*, (ii)* $T_m = \{m\}$ *if* $m \in T$ *and* $m > l$*, (iii)* $\min T_m \geq \max T_{m+1}$ *for all* $m < l$*, (iv)* $T = \cup_{m \leq l} T_m$*, and (v)* $q(m) = 1$ *and* $\hat{q}(m) = 0$ *for all* $m < l$*.*

The structure of the equilibrium strategy described by Lemma 3 is illustrated in Figure 1. In the figure, an arrow from node n to m indicates that $p(m; n) > 0$. When the number of clients of good quality is greater than the minimum number l of good ratings issued, the agency issues a truthful report with probability 1. When the number of clients of good quality is less than l, the agency exaggerates the number of good quality clients; indeed it issues more good ratings when there are fewer clients of good quality.[14] This characterization follows from the result in Lemma 2 that the credibility of a good rating decreases with the total number of good ratings, and the result in Lemma 1

[13] In a mechanism design setup in which the designer cannot fully commit to an outcome function, Bester and Strausz (2001) show that any Pareto efficient equilibrium allocation can be achieved with an equilibrium in which each type reports truthfully with positive probability. Their result requires messages to be cheap talk in a signaling setting such as the present model where there is no commitment at all, and therefore it does not apply here.

[14] When the number of clients of good quality is equal to l, the agency may tell the truth in equilibrium, or it may randomize between issuing l or issuing more than l good ratings (as depicted in Figure 1).

that the agency has a stronger incentive to inflate to a more credible signal when there are more clients of good quality. Part (iv) of Lemma 3 establishes that in any non-full support equilibrium the aggregate support of the equilibrium strategy, T, satisfies the threshold property that all signals $m \geq l$ are sent with positive probability. Finally, part (v) of the lemma specifies a unique set of out-of-equilibriums beliefs $q(m)$ and $\hat{q}(m)$ for $m \notin T$ that satisfy the refinement. It is established by showing that if the agency finds it weakly optimal to send an out-of-equilibrium signal $m < l$ when there are $n \neq m$ good quality clients, then the signal is strictly optimal when there are exactly m good quality clients.

The main result of this section is Proposition 1 below. Since the proof in the Appendix is rather involved, it is useful here to describe the main steps. We start by showing that the restrictions imposed by Lemma 3 on the structure of T_n in each state $n > 0$, together with the necessary equilibrium conditions that there are no profitable deviations, result in certain iterative constraints on the equilibrium reporting strategy $p(m;n)$ given the reporting strategies in states $n+1,\ldots,N$ (Definition A.1 and Lemma A.1). Next we show that, given the reporting strategies in state $n = 1,\ldots,N$, all the equilibrium conditions can be satisfied by choosing the value of $p(m;0)$ appropriately for each $m = 0,\ldots,N$, and an equilibrium obtains when such values satisfy $\sum p(m;0) = 1$ (Definition A.2 and Lemma A.2). Then, we show that the set of reporting strategies for state $1,\ldots,N$ satisfying the necessary conditions for an equilibrium is closed and connected (Lemmas A.3 and A.4). In the final step of the proof we show that the set of values $p(m;0)$ that completes the equilibrium conditions has the property that $\sum p(m;0)$ is continuous over the collection of reporting strategies in state $1,\ldots,N$ satisfying the necessary equilibrium conditions (Lemma A.5). We use this property to establish that an inflationary equilibrium always exists, and there is a non-full support equilibrium with $l > 0$ if $q^* < 1-\pi_0$.

Proposition 1. *An inflationary equilibrium exists under centralized rating. Further, a non-full support equilibrium exists if* $q^* < 1-\pi_0$.

The condition for the existence of a non-full support equilibrium has a simple interpretation.[15] If the likelihood π_0 that there is no client of good quality is sufficiently small, then by inflating exclusively in the state when there are no good clients, it is not possible to drive down the market beliefs $q(m)$ to q^* for each positive number of good ratings m. In this case, a randomization strategy $p(m;n)$ with non-full support and $p(0;0) = 0$ is credible. Further, more probability mass from low states (small numbers of good quality clients) may be credibly distributed to higher states through the randomization described in the proof of Proposition 1, leading to a higher threshold l. Recall that a non-full support equilibrium exists under individual rating if and only if $q^* < \pi$, which implies $q^* < 1-\pi_0$. Thus, correlated randomization under centralized rating can allow the agency to credibly apply a non-full support signaling strategy and achieve a

[15]The proof of Proposition 1 considers only one particular kind of full support equilibrium, with $l = 0$ and $p(n;n) = 1$ for all $n > 1$. There are typically multiple equilibria with $l = 0$ and $q(m) = q^*$, and they can also coexist with a non-full support equilibrium.

higher average level of credibility, when independent randomization under individual rating implies that the credibility of good ratings is fixed at q^*.

The construction of our inflationary equilibrium in Proposition 1 applies to any prior distribution of the client qualities. This is in contrast to the equilibrium construction under decentralized rating in the next section, which relies on a natural correlation structure of the quality distribution. We conclude this section by pointing out that the equilibrium structure under centralized rating generally depends both on the incentive structure as represented by properties of the payoff functions and on the correlation structure of the quality distribution, and these two factors can be disentangled only in special cases. For example, the sufficient condition for a non-full support inflationary equilibrium in Proposition 1 depends on the quality distribution captured by the probability π_0 and on the incentive to inflate represented by q^*. Under reasonable specifications of the correlation structure, an increase in quality correlation increases π_0 and thus makes it harder to satisfy the condition for fixed q^*. At the other end, the necessary and sufficient condition for the existence of an equilibrium with $l=N$ is given by

$$(N-1)U(G,g,1)+U(B,b,0)\leq (N-1)U(G,g,\pi)+U(B,g,\pi),$$

since from the proof of Proposition 1 the incentive of state $N-1$ not to deviate from reporting N to reporting $N-1$ determines the equilibrium. Whether the above condition holds depends only on the incentive structure and π, not on the correlation structure. More generally, the effects of the incentive structure and the quality correlation on the equilibrium structure cannot be studied separately.[16] In Section 5, we use a specific model of the correlation structure to give a definitive characterization of the effects of correlation when the number of clients is arbitrarily large.

4. Decentralized rating: A model of competing signals

Under decentralized rating, rating information is shared among all markets, as under centralized rating, but each client is rated by a self-interested rater of the agency with no access to the quality information of other clients.[17] We implicitly assume that as an alternative to centralized rating, the agency can limit the information about client quality available to each rater to the single client that the rater is assigned to, and at

[16]Examples can be constructed in which an increase in quality correlation can either increase or decrease the equilibrium threshold. Details are available in a supplementary file on the journal website, http://econtheory.org/supp/370/supplement.pdf.

[17]The two features of decentralized rating, namely the restricted information and independent payoffs, that set it apart from centralized rating, are jointly responsible for our comparison results regarding these two rating schemes. It is possible to consider a hybrid model in which the state vector is observed by all raters, but each rater maximizes its own payoff. In such a model an inflationary equilibrium is given by $\{p(n)\}$ such that for each n a rater with a bad quality client inflates with probability $p(n)$, and $\{q(m)\}$ such that each client rated g is believed to be good with probability $q(m)$ when the total number of clients rated g is m. In contrast to the results we establish below, in such a hybrid model $q(m)$ is not necessarily monotone in m and it is always true that for every realized n the incentives to issue a good rating are stronger for a rater with a good quality client than for one with a bad quality client. The analysis of the model is beyond the scope of this paper.

the same time tie the incentive of the rater to the agency's payoff from that client. As pointed out in the introduction, a decentralized rating scheme may also be thought of as a decentralized market structure in which each rater represents an independent rating agency. In terms of the strategy space, decentralized rating is the same as individual rating, as only independent randomization across clients is feasible. If the underlying client qualities are independently distributed, decentralized rating produces the same equilibrium outcome as does individual rating. However, since ratings information is shared among all markets, when the underlying qualities are correlated, each market can use the other ratings to make inferences about the quality of its own client.

In this section we construct an inflationary equilibrium under decentralized rating. Unlike the case of centralized rating, the analysis of decentralized rating requires a model of quality correlation across clients. In Definition 1 below, we give precise formulations for positive and negative correlations among client qualities. These formulations allow us to give sharp characterizations of inflationary equilibria: under positive (negative) correlation each rater expects a greater (lower) number of good ratings conditional on G than conditional on B, in the sense of first-order stochastic dominance, and the credibility of a good rating is increasing (decreasing) in the total number of good ratings issued. The main result of this section establishes the existence of a symmetric inflationary equilibrium under decentralized rating, and a necessary and sufficient condition for a full support equilibrium. It turns out that this condition is identical to the condition under individual rating. We postpone to the next section a discussion of how in a decentralized scheme the rating agency can gain in credibility under correlated qualities and therefore become better off relative to individual rating.

Define a random variable X_i, $i=1,\dots,N$, such that $X_i=1$ if $S_i=G$ and $X_i=0$ if $S_i=B$. Let $f(X_1,\dots,X_N)$ represent the joint probability mass function of the random vector $X=(X_1,\dots,X_N)$.

Definition 1. We say that X is *multivariate totally positive of order 2* (MTP_2) if, for all $x,y\in\{0,1\}^N$,

$$f(x\vee y)f(x\wedge y)\geq f(x)f(y),$$

where $x\vee y=(\max\{x_1,y_1\},\dots,\max\{x_N,y_N\})$ and $x\wedge y=(\min\{x_1,y_1\},\dots,\min\{x_N,y_N\})$. We say that X is *multivariate reverse rule of order 2* (MRR_2) if the inequality is reversed.

The definition of MTP_2 is the same as log-supermodularity, also referred to as affiliation. It is a commonly used concept of positive dependence among random variables in the statistics literature (see, for example, Joe 1997) and in the auction literature (see, for example, Milgrom and Weber 1982). Similarly, MRR_2 can be used to capture the idea of negative dependence among random variables. These dependence concepts are stronger than the notion of positive or negative "quadrant dependence" used by Lehmann (1966).

We focus on symmetric inflationary equilibria in which for each $i=1,\dots,N$, the common signaling strategy satisfies $\Pr[s_i=g\mid S_i=G]=1$ and $\Pr[s_i=g\mid S_i=B]=p$ for some $p\in[0,1]$.[18] Define the random variable Y_i, $i=1,\dots,N$, such that $Y_i=1$ if $s_i=g$

[18]In the proof of Proposition 2 below, we use Assumption 2 to establish that the indifference condition

and $Y_i = 0$ if $s_i = b$. Also let $Z_i = \sum_{j \neq i} X_j$ and $\tilde{Z}_i = \sum_{j \neq i} Y_j$.

Fix some $i = 1, \ldots, N$. For each $m = 1, \ldots, N$, let $r^G(m)$ be the probability of a total number m of good ratings conditional on $S_i = G$ and $s_i = g$:

$$r^G(m) = \Pr[\tilde{Z}_i = m - 1 \mid X_i = 1, Y_i = 1].$$

Similarly, let

$$r^B(m) = \Pr[\tilde{Z}_i = m - 1 \mid X_i = 0, Y_i = 1].$$

Note that $r^G(0) = r^B(0) = 0$. Intuitively, for any fixed p, under MTP_2 each individual rater expects to find more good ratings when the quality of his own client is good than when it is bad, while the reverse is true under MRR_2. This idea is formalized in the following lemma.

Lemma 4. *In any inflationary equilibrium, $\{r^G(m)\}$ first-order stochastically dominates $\{r^B(m)\}$ under MTP_2; the reverse is true under MRR_2.*

Given any inflationary equilibrium, the beliefs $q(m)$, $m = 1, \ldots, N$, are given by

$$q(m) = \Pr[X_i = 1 \mid Y_i = 1, \tilde{Z}_i = m - 1].$$

Let $\beta(t, k, p)$ represent the probability of k successes out of t Bernoulli trials with independent probability of success p; that is,

$$\beta(t, k, p) = \binom{t}{k} p^k (1 - p)^{t-k}.$$

Then, $q(m)$ can be written more explicitly as

$$q(m) = \frac{\sum_{n=0}^{m} \pi_n \beta(N - n, m - n, p) n}{m \sum_{n=0}^{m} \pi_n \beta(N - n, m - n, p)}. \tag{6}$$

This formula is valid as long as the denominator is strictly positive, which happens if $p < 1$. We refer to an inflationary equilibrium with $p < 1$ as a full support equilibrium.

Lemma 5. *In any full support inflationary equilibrium, $q(m)$ is increasing in m under MTP_2 and is decreasing in m under MRR_2.*

This result is quite intuitive. In an inflationary equilibrium the perception of a good rating depends on the total number of good ratings in all markets: the perception improves with more good ratings when the client qualities are positively correlated, and it deteriorates when the qualities are negatively correlated. We are now ready to use Lemmas 4 and 5 to establish existence of an inflationary equilibrium. Note that in any inflationary equilibrium, $\hat{q}(m) = 0$ for all $m = 0, \ldots, N - 1$.

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between g and b under B is sufficient to imply truth-telling under G. But this result presumes the signaling structure of inflationary equilibria. The single crossing condition of Assumption 2 is generally insufficient to rule out deflationary strategies.

Proposition 2. *There exists an inflationary equilibrium under decentralized rating. Further, if $\pi < q^*$, there is a full support inflationary equilibrium.*

Proof. A necessary and sufficient condition for the existence of a full support inflationary equilibrium is that there exists $p \in (0,1)$ such that (i) $s_i = g$ is weakly preferred to $s_i = b$ if $S_i = G$:

$$\sum_{m=1}^{N} r^G(m) U(G, g, q(m)) \geq U(G, b, 0)$$

and (ii) $s_i = g$ and $s_i = b$ yield the same expected payoff if $S_i = B$:

$$\sum_{m=1}^{N} r^B(m) U(B, g, q(m)) = U(B, b, 0). \tag{7}$$

Under MTP$_2$, Lemma 4 states that $\{r^G(m)\}$ first-order stochastically dominates $\{r^B(m)\}$, while Lemma 5 states that $q(m)$ is increasing m. Therefore,

$$\sum_{m=1}^{N} r^G(m) U(G, g, q(m)) \geq \sum_{m=1}^{N} r^B(m) U(G, g, q(m)).$$

It follows from Assumption 2 that condition (ii) implies condition (i). Under MRR$_2$, $\{r^B(m)\}$ first-order stochastically dominates $\{r^G(m)\}$ while $q(m)$ is decreasing m, so again condition (ii) implies condition (i) by Assumption 2. Now, consider the indifference condition (ii). If $p = 0$, we have $q(m) = 1$ for all $m = 1, \ldots, N$. By Assumption 3,

$$\sum_{m=1}^{N} r^B(m) U(B, g, q(m)) > U(B, b, 0)$$

when $p = 0$. If $p = 1$, we have $q(N) = \sum_n \pi_n n/N = \pi$ and the left-hand-side of condition (ii) becomes $U(B, g, \pi)$. Under Assumption 2, the refinement implies that the out-of-equilibrium belief $\hat{q}(N-1)$ is equal to 0. Thus, if $U(B, g, \pi) < U(B, b, 0)$, or equivalently $\pi < q^*$, then by the intermediate value theorem there exists $p \in (0,1)$ such that the equilibrium condition (ii) is satisfied, and hence there is a full support inflationary equilibrium. If instead $\pi \geq q^*$, with the out-of-equilibrium belief $\hat{q}(N-1)$ set to 0, g is weakly preferred to b under quality B, implying that g is strictly preferred to b under G. We thus have a non-full support equilibrium with $p = 1$. □

The condition for the existence of a full support inflationary equilibrium is identical to the condition for the existence of the unique full support inflationary equilibrium under individual rating. This is perhaps not surprising, because the only difference between decentralized rating and individual rating is informational, in that the market for each client observes the ratings for other clients as well as its own and can use these ratings to make inferences about the quality of the client it cares about. Such a difference is not relevant in a non-full support equilibrium of decentralized rating, as the markets always observe a total of N good ratings. The informational difference between decentralized rating and individual rating is similarly irrelevant with independent client qualities. In that case, we have $\pi_n = \beta(N, n, \pi)$ for each $n = 0, \dots, N$. Then, for each $m = 1, \dots, N$, direct calculations reveal that

$$q(m) = \frac{\sum_{n=1}^{m} (\pi/(1-\pi)p)^n (1/((m-n)!(n-1)!))}{m \sum_{n=0}^{m} (\pi/(1-\pi)p)^n (1/((m-n)!n!)} = \frac{\pi}{\pi + (1-\pi)p}.$$

Thus, under independence, decentralized rating reduces to individual rating.

The equilibrium probability of ratings inflation is determined by the indifference condition (7). Generally both the incentive structure as represented by the payoff function $U(B, g, \cdot)$ and the entire client quality distribution affect the equilibrium behavior. As under centralized rating, increasing quality correlation does not always have the same effects on the equilibrium probability of inflation. In the next section, using a specific model of the correlation structure and considering the limit case when the number of clients is arbitrarily large, we can characterize the effects of correlation on the equilibrium behavior.

5. Comparing rating schemes: Credibility and welfare

A comparison between centralized rating and decentralized rating in terms of the equilibrium credibility of good ratings and the ex ante payoffs to the agency generally depends on the the underlying correlation structure. In Proposition 1, we establish that there always exists an inflationary equilibrium under centralized rating that does at least as well as the full support inflationary equilibrium under individual rating. Moreover, when $q^* < 1 - \pi_0$, there is a non-full support equilibrium that does strictly better. This condition is rather weak, and is easily satisfied when the qualities are independently distributed, as long as N is not too small. In contrast, with independently distributed qualities, the unique inflationary equilibrium under decentralized rating is payoff-equivalent to the full support inflationary equilibrium under individual rating. Thus, we expect centralized rating to dominate decentralized rating for the agency when there is weak correlation among the qualities.

The next set of results shows that both the equilibrium credibility of good ratings and the ex ante payoff to the agency under decentralized rating improve relative to the benchmark of individual rating when the qualities are correlated. First, we introduce a definition of equilibrium credibility of good ratings under decentralized rating. For any p (the probability of inflating), consider the expression

$$\sum_{n=0}^{N} \frac{n\pi_n}{N\pi} \sum_{m=n}^{N} \beta(N-n, m-n, p) q(m). \tag{8}$$

This expression may be thought of as an average measure of credibility of good ratings under decentralized rating, as the credibility of a single given good rating depends on the total number of good ratings. It is an average across states, with each state weighted both by the prior probability of the state and by the number of good quality clients in the state. Under individual rating, the same expression (8) applies, but $q(m)$ is constant and equal to q because the markets are separate. Since $\sum_{m=n}^{N}\beta(N-n,m-n,p)=1$, the above definition of credibility is consistent with the definition given under individual rating. Further, if we replace $\beta(N-n,m-n,p)$ with $p(m;n)$ in (8), we have a measure of credibility under centralized rating.

Under decentralized rating, as under individual rating, the definition (8) of credibility, and correspondingly the expression with $p(m;n)$ replacing $\beta(N-n,m-n,p)$ for centralized rating, corresponds one-to-one with the average expected loss of the N markets, and one-to-one with the expected average marginal value of information provided by the ratings in equilibrium. To see this, note that the total expected loss in equilibrium is given by

$$\sum_{n=0}^{N}\pi_n\sum_{m=n}^{N}\beta(N-n,m-n,p)(n(1-q(m))^2+(m-n)q^2(m)).$$

From equation (6) we have

$$\sum_{n=0}^{m}\beta(N-n,m-n,p)q(m)m=\sum_{n=0}^{m}\pi_n\beta(N-n,m-n,p)n,$$

and therefore by equation (1) the total loss is equal to $N\pi(1-Q)$, where Q is our credibility measure given by equation (8). Recall that the average expected loss in a full support equilibrium under individual rating is $\pi(1-q^*)$, and is $\pi(1-\pi)$ in the absence of any ratings information. Thus, the credibility measure given by (8) corresponds one-to-one with the expected average marginal value of information provided by the ratings in equilibrium.

To compare equilibrium credibility between decentralized rating and individual rating, we first note that

$$\begin{aligned} r^G(m) &= \sum_{n=1}^{N}\Pr[\tilde{Z}_i=m-1 \mid X_i=1,Y_i=1,Z_i=n-1]\Pr[Z_i=n-1 \mid X_i=1,Y_i=1] \\ &= \sum_{n=1}^{m}\beta(N-n,m-n,p)\frac{n}{N}\frac{\pi_n}{\pi}. \end{aligned}$$

Hence the credibility measure is simply

$$\sum_{n=0}^{N}\frac{n\pi_n}{N\pi}\sum_{m=n}^{N}\beta(N-n,m-n,p)q(m)=\sum_{m=1}^{N}r^G(m)q(m).$$

Thus, the average measure of the credibility of good ratings under decentralized rating is equal to the market belief expected by a rater with a good quality client. Next, recall that under individual rating, the market's belief upon observing g is given by $\pi/(\pi+(1-\pi)p)$ if p is the probability that rating g is issued under quality B. The next lemma captures the idea that for any probability of ratings inflation, under decentralized rating correlation across client qualities imposes a discipline on incentives to inflate by making it harder for each individual rater to fool its own market.

Lemma 6. *Under decentralized rating, for any $p<1$,*

$$\sum_{m=1}^{N} r^B(m)q(m) \leq \frac{\pi}{\pi+(1-\pi)p}.$$

This result is the key to our comparison results below. Under decentralized rating the weighted average of the market belief conditional on a bad quality client is lower than the market belief under individual rating for the same probability of rating inflation. That is, a rater that issues an inflated rating on a bad quality client expects on average a less favorable market belief under either positive or negative correlation than when qualities are statistically independent. The intuition is that under either positive correlation (MTP_2) or negative correlation (MRR_2), for the rater with a bad quality client, the weights are smaller for higher market beliefs q, so that the weighted average is lower than the average when the qualities are independently distributed for independent randomizations with the same probability of inflation p. For example, under positive correlation, a higher market belief q is associated with a greater number of good ratings, but since a rater with a bad quality client expects statistically fewer good quality clients and thus fewer good ratings, a higher market belief receives a smaller weight.

Our first comparison result is that when $U(B,g,q)$ is weakly concave in q, then at any full support inflationary equilibrium under decentralized rating, the equilibrium probability of inflation is lower than the full support equilibrium probability of inflation under individual rating.[19] Furthermore, under the same concavity condition, the equilibrium credibility is higher under decentralized rating than it is under individual rating.

Proposition 3. *Suppose $U(B,g,q)$ is concave in q. In any full support inflationary equilibrium under decentralized rating, the probability of inflation is lower and the credibility is higher than in the full support inflationary equilibrium under individual rating.*

Proof. In a full support inflationary equilibrium, for each $i=1,\ldots,N$, we must have the indifference condition between $s_i=g$ and $s_i=b$. This condition gives

$$\sum_{m=1}^{N} r^B(m)U(B,g,q(m)) = U(B,g,q^*). \tag{9}$$

Since $U(B,g,q)$ is concave in q, we have

$$U\left(B,g,\sum_{m=1}^{N} r^B(m)q(m)\right) \geq U(B,g,q^*). \tag{10}$$

It then follows from Lemma 6 that

$$U(B,g,\pi/(\pi+(1-\pi)p)) \geq U(B,g,q^*),$$

[19]This comparison between equilibrium probabilities of inflation under decentralized and individual rating holds so long as the function is not too convex.

where p is the equilibrium probability of inflation. Comparing this inequality to equation (5) in a full support inflationary equilibrium under individual rating, we immediately obtain that the equilibrium probability of inflation is lower under decentralized rating than it is under individual rating.

By Lemmas 4 and 5, $\{r^G(m)\}$ first-order stochastically dominates $\{r^B(m)\}$ and $q(m)$ is increasing under MTP$_2$, while $\{r^B(m)\}$ stochastically dominates $\{r^B(m)\}$ and $q(m)$ is decreasing under MRR$_2$. In either case, we have

$$\sum_{m=1}^{N} r^G(m)q(m) \geq \sum_{m=1}^{N} r^B(m)q(m).$$

From inequality (10) we then have

$$\sum_{m=1}^{N} r^G(m)q(m) \geq q^*,$$

implying that the equilibrium credibility is higher under decentralized rating. □

For welfare comparison between decentralized rating and individual rating, we say that $U(B,g,\cdot)$ is "more concave" than $U(G,g,\cdot)$ if there is a weakly concave function H such that $U(B,g,q) = H(U(G,g,q))$. We have a second comparison result, as follows.

Proposition 4. *Suppose $U(B,g,q)$ is more concave in q than $U(G,g,q)$. Then, the agency's payoff in a full support inflationary equilibrium under decentralized rating is higher than it is in the full support inflationary equilibrium under individual rating.*

Proof. If $U(B,g,q)$ is more concave in q than $U(G,g,q)$, the indifference condition (9) implies

$$\sum_{m=1}^{N} r^B(m)U(G,g,q(m)) \geq U(G,g,q^*).$$

Under MTP$_2$, $\{r^G(m)\}$ first-order stochastically dominates $\{r^B(m)\}$ and $q(m)$ is increasing, and so

$$\sum_{m=1}^{N} r^G(m)U(G,g,q(m)) \geq U(G,g,q^*).$$

Under MRR$_2$, $q(m)$ is decreasing but $\{r^B(m)\}$ stochastically dominates $\{r^B(m)\}$, so again the inequality is true. □

Under decentralized rating each client i is exposed to a greater risk when $S_i = G$ than it is under individual rating, because of the uncertainty regarding the ratings of other clients. However, the beliefs are more favorable under G than they are under B in the sense of first-order stochastic dominance. Thus, welfare improves as long as the agency is not too much more risk-averse when $S_i = G$ than when $S_i = B$.

Since the strategy space under decentralized rating is the same as it is under individual rating, the above results show that the gains in credibility and welfare under decentralized rating come from sharing ratings information among the markets. We expect that the gains are larger when the correlation is stronger. Indeed, the next proposition establishes that when the correlation across client qualities is almost perfect, there is a limit inflationary equilibrium with "truth-telling," i.e., the equilibrium probability of inflation converges to 0. Let $\{\pi_0^k,\ldots,\pi_N^k\}$ be a sequence of probability distributions that satisfy MTP_2, such that (i) $\lim_{k\to\infty}\sum_{n=1}^{N-1}\pi_n^k=0$ and (ii) $\lim_{k\to\infty}\pi_N^k/(\pi_N^k+\pi_0^k)<q^*$. The first condition means that the states become almost perfectly positively correlated as k becomes arbitrarily large. The second condition guarantees that there exists no pooling equilibrium with $p(N;n)=1$ for all n when k is large.

Proposition 5. *Under decentralized rating, truth-telling is a limit inflationary equilibrium when k goes to infinity.*

Proof. Equation (7) is necessary and sufficient for an inflationary equilibrium under decentralized rating. As in the proof of Proposition 2, for $p=0$, the left-hand side of (7) is strictly larger than the right-hand side for any k. Next, for all $p>0$, the limit of left-hand side as k goes to infinity is strictly less than

$$p^{N-1}U(B,g,1)+(1-p^{N-1})U(B,g,0).$$

This is because in the limit when the correlation is perfect, from equation (6) we have $q(m)=0$ for all $m<N$, while $q(N)<1$. Let $\tilde{p}$ be the value of p that solves

$$p^{N-1}U(B,g,1)+(1-p^{N-1})U(B,g,0)=U(B,b,0).$$

Then, for all $0<p<\tilde{p}$, the limit of the left-hand side of (7) as k goes to infinity is strictly smaller than the right-hand side. Hence, for each p there exists $k(p)$ such that for all $k>k(p)$ there is an inflationary equilibrium with the probability of inflation strictly between 0 and p. Since this construction of $\tilde{p}$ and $k(p)$ holds for all p, by taking p arbitrarily close to 0 we can establish truth-telling (i.e., $p=0$) as a limit point of a sequence of inflationary equilibria for k going to infinity. □

While strong correlation enhances credibility and improves welfare under decentralized rating, the opposite is true under centralized rating. To see this, note that the conditions made on the convergence of the sequence of the distributions π^k imply that in the limit of k going to infinity, there is no non-full support equilibrium by Proposition 1. Thus, centralized rating cannot improve upon individual rating when correlation is almost perfect. Correlation of the underlying qualities reduces the manipulation room under both decentralized rating and centralized rating. Under decentralized rating the constraint imposed by correlation makes it harder for a rater to fool the market with a good rating, and forces the individual raters to tone down the exaggeration. This then results in a greater ex ante payoff relative to individual rating. In contrast, strong correlation makes correlated randomization under centralized rating less effective.

5.1 *Correlation, credibility, and welfare: An example*

For an analysis involving non-extreme values of correlation, the notion of quality correlation is ambiguous, and a more specific description of the multivariate probability distribution is required. We illustrate how quality correlation affects the equilibrium behavior and the welfare properties of centralized rating versus decentralized rating using the following simple model of correlation. With probability α, the client qualities are perfectly correlated, in which case π is the probability that all clients have good quality and $1-\pi$ is the probability that all have bad quality; with probability $1-\alpha$, the client qualities are independent, and π is the probability that each client is of good quality. In this model, the quality distribution satisfies MTP$_2$, and the parameter α measures the degree of correlation. When the number of clients is arbitrarily large, the quality distribution converges to three mass points: with probability $\alpha\pi$ all clients have good quality, with probability $\alpha(1-\pi)$ no client has good quality, and with the remaining probability $1-\alpha$ a fraction arbitrarily close to π of all clients have good quality. We analyze the limit case of N going to infinity for centralized rating and decentralized rating separately before comparing the two schemes.

Centralized rating We assume that $q^* < 1-\alpha(1-\pi)$; otherwise, in the limit case the equilibrium outcome is equivalent to the full support equilibrium under individual rating with all beliefs given by q^*. Further, we assume that for all $\mu \in (\pi, 1]$,

$$\pi U(G,g,1)+(\mu-\pi)U(B,b,0) > \pi U\left(G,g,\pi/\mu\right)+(\mu-\pi)U\left(B,g,\pi/\mu\right). \tag{11}$$

In words, the agency strictly prefers rating truthfully to issuing any higher fraction μ of good ratings when it is common knowledge that with probability 1 a fraction π of the clients is of good quality.[20]

Claim 1. *There is a unique limit threshold fraction λ such that when all clients have bad quality the limit equilibrium message λ is either equal to π or strictly below it.*

Using equation (11), in the appendix (Lemma A.6) we formally state and establish that in any equilibrium there is approximate truth-telling as long as it is not the case that all clients have bad quality. Intuitively, for any $\alpha < 1$, because in the limit the probability mass of all states other than $n=0$ and $n=N$ is concentrated around $N\pi$, if in these states the agency issues a fraction μ of good ratings bounded away from π, then the market belief upon observing μ cannot exceed π/μ in the limit. This inflation also implies that the market belief upon observing a fraction close to π of good ratings approaches 1, which is inconsistent with equilibrium by condition (11). As a result, the

[20] If this assumption is violated, truth-telling does not obtain even when the qualities are independent ($\alpha = 0$) and the number of clients is arbitrarily large, so that the market beliefs depend on the fraction of observed good ratings, and not on the signaling strategy. In this case, ratings inflation occurs for reasons unrelated to the issue of credibility. If the markets could jointly commit to their actions, then truth-telling in the limit of $\alpha = 0$ and N going to infinity can easily be obtained, by having the markets assign the belief of 1 to exactly a fraction π of all clients and a belief of 0 to the rest. This observation is an example of the general result given by Jackson and Sonnenschein (2007), in which case assumption (11) is not needed.

equilibrium threshold fraction l^N/N cannot be strictly above π in the limit. Which case arises depends only on the incentives to inflate when all client qualities are bad.

Case 1: $\lambda=\pi$. If

$$U(B,g,\pi)>\pi U(B,g,1)+(1-\pi)U(B,b,0), \tag{12}$$

then in state $n=0$ the agency strictly prefers issuing all good ratings to pooling with states close to $N\pi$, implying $\lambda=\pi$. In this case, changes in α have no effect on the equilibrium behavior, but since the agency reports truthfully when there is a fraction π of good quality clients, the equilibrium outcome approaches truth-telling as α goes to 0.[21]

Case 2: $\lambda<\pi$. If the inequality (12) is reversed, then under the beliefs in (12), in state $n=0$ the agency strictly prefers to pool with states close to $N\pi$. This is not an equilibrium, as the beliefs upon observing the messages close to $N\pi$ fall below 1, leading to deviations in states close to $N\pi$, because by Lemma 3 the belief upon observing out-of-equilibrium messages just below $N\pi$ is 1. Instead, in state $n=0$ the agency is indifferent among issuing three different fractions of good ratings, λ, π, and 1, and distributes the probability mass $\alpha(1-\pi)$ between the latter two fractions. The value of $\lambda<\pi$ and the probability mass $\theta>0$ assigned to the fraction π are uniquely determined by the indifference conditions:[22]

$$\begin{aligned}&\pi U\big(B,g,(1-\alpha)/(1-\alpha+\alpha(1-\pi)\theta)\big)+(1-\pi)U(B,b,0)\\&\qquad=\lambda U(B,g,1)+(1-\lambda)U(B,b,0)=U\big(B,g,\pi/(\pi+(1-\pi)(1-\theta))\big).\end{aligned} \tag{13}$$

In this case, as α increases, the equilibrium value of θ decreases, because a greater probability mass $\alpha(1-\pi)$ in state $n=0$ to be allocated between the fraction of π and the fraction of 1 depresses the market belief upon observing a fraction π of good ratings but not the belief upon observing all good ratings. As a result, the equilibrium belief upon observing all good ratings decreases, implying that the equilibrium threshold fraction λ must decrease to restore the indifference in state $n=0$ between issuing all good ratings and issuing a fraction λ of good ratings. Thus, as in the case of $\lambda=\pi$, a greater degree of correlation reduces all equilibrium beliefs. Regardless of α, centralized rating strictly improves on individual rating: if $q^*\in(\pi,1-\alpha(1-\pi))$, then we have a full support equilibrium with belief q^* under individual rating while a non-full support equilibrium with beliefs strictly larger than q^* under centralized rating; if instead $q^*<\pi$, then we have a non-full support equilibrium under individual rating with belief equal to π, while under centralized rating the beliefs are uniformly higher because the belief upon observing all good ratings is strictly larger than π for $\theta>0$.

[21] The inequality (12) implies that $q^*<\pi$, and therefore under individual rating we have a non-full support equilibrium. Since in state $n=0$ the agency issues all good ratings with probability 1, centralized rating yields the same payoff in state $n=0$ and state $n=N$ as under individual rating. The comparison between the schemes depends only on what happens when there is a fraction π of good quality clients.

[22] The value of λ determined by these conditions is strictly positive because by assumption $q^*<1-\alpha(1-\pi)$.

Decentralized rating We assume that $q^* > \pi$; otherwise, the equilibrium has non-full support as under individual rating. In a full support equilibrium, the equilibrium probability of inflation is $p < 1$, which implies that the markets can perfectly infer the state from the realized fraction of good ratings. From each individual rater's point of view, conditional on having a bad quality client, the probability that the state is $n = 0$ is $\alpha(1-\pi)/(1-\alpha\pi)$, and with the complementary probability there is a fraction arbitrarily close to π of good quality clients.

Case 1: $p = 0$. If

$$\frac{\alpha(1-\pi)}{1-\alpha\pi}U(B,g,0)+\frac{1-\alpha}{1-\alpha\pi}U(B,g,1)<U(B,b,0), \tag{14}$$

then in the limit we have $p=0$ and truth-telling. An increase in α makes this inequality more likely to hold, but otherwise has no effect on the equilibrium behavior.

Case 2: $p > 0$. If the opposite of (14) holds, we have $p > 0$ in the limit and it uniquely satisfies

$$\frac{\alpha(1-\pi)}{1-\alpha\pi}U(B,g,0)+\frac{1-\alpha}{1-\alpha\pi}U\left(B,g,\pi/(\pi+(1-\pi)p)\right)=U(B,b,0). \tag{15}$$

As α increases, the equilibrium value of p decreases. This is because with a greater conditional probability that all clients have bad quality, a good rating becomes less credible, and the probability of inflation must decrease to restore the indifference condition. As a result, an increase in α improves the equilibrium belief upon inferring that there is a fraction π of good quality clients. When α becomes large, the indifference condition can no longer hold, and we have $p=0$ and truth-telling in equilibrium. On the other hand, when α goes to 0, the equilibrium value of p is such that the equilibrium belief upon inferring that the fraction of good quality clients is π converges to q^*, which gives the same outcome as individual rating.

Centralized vs. decentralized rating To make payoff and welfare comparisons between centralized and decentralized rating schemes as α varies, we note that under either individual rating or truth-telling, the payoff to the agency and the payoff loss to the markets as represented by our credibility measure (8) are invariant to α. Under centralized rating, if $q^* \geq 1-\alpha(1-\pi)$, the equilibrium payoff is $\pi U(G,g,q^*)+(1-\pi)U(B,b,0)$ and the credibility is q^*, the same as in the full support equilibrium under individual rating. If instead $q^* < 1-\alpha(1-\pi)$, the equilibrium payoff to the agency is

$$\begin{aligned}\alpha(1-\pi)U\left(B,g,\pi/(\pi+(1-\pi)(1-\theta))\right)+\alpha\pi U\left(G,g,\pi/(\pi+(1-\pi)(1-\theta))\right)\\ +(1-\alpha)\left(\pi U\left(G,g,(1-\alpha)/(1-\alpha+\alpha(1-\pi)\theta)\right)+(1-\pi)U(B,b,0)\right)\end{aligned}$$

and the credibility measure (8) becomes

$$(1-\alpha)\frac{1-\alpha}{1-\alpha+\alpha(1-\pi)\theta}+\alpha\frac{\pi}{\pi+(1-\pi)(1-\theta)},$$

where $\theta = 0$ if (12) is satisfied, and is given by (13) otherwise. Under decentralized rating, if $q^* \le \pi$, the equilibrium payoff is $\pi U(G,g,\pi) + (1-\pi)U(B,g,\pi)$, and the credibility is π, the same as in the non-full support equilibrium under individual rating. If instead $q^* > \pi$, then the equilibrium payoff to the agency is

$$\pi\left(\frac{1-\alpha}{1-\alpha(1-\pi)}U\left(G,g,\pi/(\pi+(1-\pi)p)\right) + \frac{\alpha\pi}{1-\alpha(1-\pi)}U(G,g,1)\right) + (1-\pi)U(B,b,0)$$

and the credibility measure becomes

$$\frac{1-\alpha}{1-\alpha(1-\pi)}\frac{\pi}{\pi+(1-\pi)p} + \frac{\alpha\pi}{1-\alpha(1-\pi)},$$

where p is the equilibrium probability of inflation given by (15).

Comparison case 1: $q^* \le \pi$. In this case, decentralized rating is equivalent to individual rating, but centralized rating does better both in terms of payoff to the agency and credibility. When α approaches 0, centralized rating achieves the first best; as α increases, both the payoff to the agency and the credibility of good ratings decrease; and as α approaches 1, the equilibrium outcome of centralized rating approaches the non-full support equilibrium of individual rating, at which point centralized rating coincides with decentralized rating.

Comparison case 2: $q^* > \pi$. In this case, as α approaches 0, again centralized rating achieves the first best while decentralized rating approaches the full support equilibrium of individual rating. As α increases, the payoff to the agency and credibility decrease under centralized rating but increase under decentralized rating. When α is sufficiently large, centralized rating becomes equivalent to the full support equilibrium of individual rating, while as α approaches 1, decentralized rating achieves the first best outcome of truth-telling.

6. Concluding remarks

Providers of information often care about the way their information is used. The desire to create favorable beliefs about its clients may cause a rating agency to inflate its assessment of the quality of its clients. The exuberant stock recommendations made during the internet boom and the failure of auditors to raise alerts in a number of recent corporate scandals have heightened the public's concern about the potential conflict of interests inherent in situations where raters are advocates for the rated. Moore et al. (2005) study this kind of problem and its possible solutions from a variety of perspectives. Gentzkow and Shapiro (2006) study how competition and the concern for reputation may constrain biased reporting by the mass media. Chan et al. (2007) use a signaling model to understand why grades in academia tend to be exaggerated. None of these papers, however, examines how the credibility of ratings can be improved by coordinating or decentralizing the rating decisions, which is the main focus of our paper.

In the literature on reputational cheap talk, a bad sender type may provide useful information to the receiver to establish credibility as a good sender type so as to extract future surplus (Sobel 1985, Benabou and Laroque 1992, Morris 2001, Morgan and Stocken 2003). This effect arises in a cheap talk game where the sender has private information on both the relevant state-of-the-world and his personal bias. As a costly signaling model of credibility, our model of individual rating has a single source of private information. The equilibrium credibility of a good rating is quantifiable in our model and corresponds one-to-one with the welfare of the rating agency. These features make our model of credibility a natural benchmark for comparisons with centralized and decentralized rating schemes.

This paper is related to the small literature on multi-dimensional signaling (Quinzii and Rochet 1985, Engers 1987). This literature focuses on the conditions under which separation of types occurs. Technically, the models in the existing literature are concerned with multi-dimensional private information for the sender and one-dimensional signals. Our signaling model of centralized rating assumes exchangeability of the components of the state vector, so that the private information is the number of good clients, which is one-dimensional. However, the signal space is multi-dimensional, as a strategy specifies a number of good ratings for each number of good clients. As a result, the single crossing condition in the benchmark case of individual rating is not completely effective in either centralized rating or decentralized rating. This feature complicates the analysis but enriches the comparison analysis for the different schemes of rating. Chakraborty and Harbaugh (2007) show that in a cheap talk game where a sender and receiver interact on several unrelated issues, the sender can credibly communicate to the receiver the ranking of the private signals even if the conflicts between them are too great to permit credible communication of the signal on any single issue.[23] Their result has the interpretation that bundling independent reports may help information transmission, which is related to our result for centralized rating. However, their result follows from the observation that the sender has no incentive to deceive the receiver about the ranking of the signals, while our analysis is based on the coordination of reports in a costly signaling model.

In the literature on signaling games, a few models involve multiple senders (Bagwell and Ramey 1991, Hertzendorf and Overgaard 2001). In these models, the senders know each other's types and interact with each other directly through their signals. In contrast, the raters in our model of decentralized rating have private information about their own types and have no direct interaction except that their signals are jointly used by the receivers to make inferences about the types of the senders. Our model of decentralized rating is therefore a model of competing signals, rather than a model of competing senders.

[23] The idea that linking decisions can be payoff-improving appears also in the literature on bundling in monopoly pricing (Adams and Yellen 1976, McAfee et al. 1989) and incentive design (Maskin and Tirole 1990, Jackson and Sonnenschein 2007).

Appendix

Lemma 1. *(i) For any $m \le n < n' \le m'$, if $W(m';n) \ge W(m;n)$ then $W(m';n') > W(m;n')$; (ii) for any $n < n' \le m, m'$ and $q(m') > q(m)$, if $W(m';n) \ge W(m;n)$, then $W(m';n') > W(m;n')$; and (iii) for any $m, m' \le n' < n$ and $\hat{q}(m') > \hat{q}(m)$, if $W(m';n) \ge W(m;n)$, then $W(m';n') > W(m;n')$.*

Proof. (i) The difference of differences $[W(m';n') - W(m;n')] - [W(m';n) - W(m;n)]$ is $(n'-n)$ times

$$[U(G,g,q(m')) - U(G,b,\hat{q}(m))] - [U(B,g,q(m')) - U(B,b,\hat{q}(m))],$$

which is positive by equation (2).

(ii) The difference between $W(m';n') - W(m;n')$ and $W(m';n) - W(m;n)$ is $(n'-n)$ times

$$[U(G,g,q(m')) - U(G,g,q(m))] - [U(B,g,q(m')) - U(B,g,q(m))],$$

which is positive by Assumption 2 since $q(m') > q(m)$.

(iii) The difference between $W(m';n') - W(m;n')$ and $W(m';n) - W(m;n)$ is $(n-n')$ times

$$[U(B,b,\hat{q}(m')) - U(B,b,\hat{q}(m))] - [U(G,b,\hat{q}(m')) - U(G,b,\hat{q}(m))],$$

which is positive by Assumption 2 since $q(m') > q(m)$. □

Proof of Lemma 2. Let $\overline{m}_1 = N$, and iteratively define $\underline{m}_k$ as the smallest integer such that $\{\underline{m}_k, \dots, \overline{m}_k\} \subseteq T$, and $\overline{m}_{k+1}$ as the largest integer smaller than $\underline{m}_k$ such that $\overline{m}_{k+1} \in T$. We have the following claims regarding equilibrium and out-of-equilibrium beliefs.

1. In any inflationary equilibrium, if $q(m) < q^*$ for some $m \in T$ and $m-1 \in T$, then $q(m-1) < q(m) < q^*$. Otherwise, $W(m-1;n) > W(m;n)$ for all $n \le m-1$, and either $m \notin T$ or $q(m) = 1$, a contradiction in either case.

2. If $q(m) < q(m')$ for all $m, m' \in \{\underline{m}_k, \dots, \overline{m}_k\}$ and $m < m'$, then $p(n;n) < 1$ for all $n \in \{\underline{m}_k, \dots, \overline{m}_k - 1\}$. Otherwise, $W(n;n) \ge W(n+1;n)$ implies $W(n;n') > W(n+1;n')$ for all $n' < n$ by part (ii) of Lemma 1, implying that either $n+1 \notin T$ or $q(n+1) = 1$, a contradiction in either case.

3. For any $x \notin T$, if for each k such that $\underline{m}_k > x$ we have $q(m) < q(m') < q^*$ for all $m, m' \in \{\underline{m}_k, \dots, \overline{m}_k\}$ and $m < m'$, then the out-of-equilibrium belief is $\hat{q}(x) = 0$. Suppose instead $\hat{q}(x) > 0$. We show that $W(x;n) \ge W(t_n;n)$ for any $n > x$ and $t_n \in T_n$ implies that $W(x;n-1) > W(t_{n-1};n-1)$ for any $t_{n-1} \in T_{n-1}$. An iteration of this result leads to $\hat{q}(x) = 0$ by the refinement, a contradiction that establishes the claim. For any $n > x$, there are two cases. In the first case, either there is no k with $n = \overline{m}_k + 1$, which by the previous claim implies that there exists a $t_{n-1} \in T_{n-1}$ such that $t_{n-1} \ge n$, or $n = \overline{m}_k + 1$ for some k but $p(n-1;n-1) < 1$, which implies again that there exists $t_{n-1} \in T_{n-1}$ such that $t_{n-1} \ge n$. Then, since

$W(x;n) \geq W(t_n;n)$, we have $W(x;n) \geq W(t_{n-1};n)$ by optimality, which implies $W(x;n-1) > W(t_{n-1};n-1)$ by part (i) of Lemma 1. In the second case, $n = \overline{m}_k + 1$ for some k and $p(n-1;n-1) = 1$. Since $\hat{q}(x) > 0$ and $\hat{q}(n-1) = 0$, by part (iii) of Lemma 1, $W(x;n) \geq W(n-1;n)$ implies that $W(x;n-1) > W(n-1;n-1)$.

4. If for some $\overline{m}_k > 0$ we have $q(m) < q^*$ for all $m > \overline{m}_k$, then $q(\overline{m}_k) < q^*$. To see this, suppose that $q(\overline{m}_k) \geq q^*$. By construction $\overline{m}_k + 1 \notin T$ and by the previous claim, $\hat{q}(\overline{m}_k + 1) = 0$. Next, for any $n \leq \overline{m}_k$ and any $t_n \in T_n$, $W(\overline{m}_k + 1;n) \geq W(t_n;n)$ implies $W(\overline{m}_k + 1;n) \geq W(\overline{m}_k;n)$. Since $\hat{q}(\overline{m}_k + 1) = 0$, and $q(\overline{m}_k) \geq q^*$ by assumption, $W(\overline{m}_k + 1;n) \geq W(\overline{m}_k;n)$ implies $q(\overline{m}_k + 1) \geq q^*$. It follows that $W(\overline{m}_k + 1;\overline{m}_k + 1) > W(t_{\overline{m}_k+1};\overline{m}_k + 1)$ for any $t_{\overline{m}_k+1} \in T_{\overline{m}_k+1}$. The refinement then implies $q(\overline{m}_k + 1) = 1$, a contradiction.

Using these four claims, we now establish that in any equilibrium $q(m) \geq q^*$ for all $m \in T$. Suppose instead $q(m) < q^*$ for some $m \in T$. Then, $q(N) < q^*$; otherwise, $W(N;n) > W(m;n)$ for all $n \leq m$, contradicting the assumption that $m \in T$. Claims 1 and 4 above then imply that $q(m) < q^*$ for all $m \in T$ and $m > 0$. If $l > 0$, we have $W(0;0) > W(m;0)$ for all $m \in T$ regardless of $\hat{q}(0)$, a contradiction. If $l = 0$ and $1 \in T$, we have $p(0;0) = 1$ and $q(1) = 1$, again a contradiction. Finally, if $l = 0$ and $1 \notin T$, since $\hat{q}(1) = 0$ by claim (3) above and $q(t_1) < q^*$ for any $t_1 \in T_1$, we have $W(1;1) > W(t_1;1)$ whenever $W(1;0) \geq W(0;0)$, which implies $q(1) = 1$ by the refinement, again a contradiction.

(i) For the first part of the lemma, note that if $q(m) > q^*$ for some $m > 0$ and $m \in T$, then $W(m;0) > W(0;0)$. This contradicts the assumption that $l = 0$. Thus, $q(m) = q^*$ for all $m > 0$ such that $m \in T$.

(ii) For the second part, note that if $q^* \leq q(m') < q(m)$ or if $q^* < q(m') = q(m)$ for some $m, m' \in T$, and $m > m'$, we have $W(m;n) > W(m';n)$ for all $n \leq m'$, contradicting the assumption that $m' \in T$. Thus, it remains to prove that if $q(m) = q^*$ for some $m \in T$, then $q(m') = q^*$ for all $m \in T$ and $0 < m' < m$. To establish this last claim, suppose $q(m) = q^*$ for some $m \in T$. There are two cases. First suppose $m - 1 \in T$. If $q(m-1) > q^*$, then $W(m-1;n) > W(m;n)$ for all $n \leq m-1$. This implies $q(m) = 1$, a contradiction. Thus $q(m-1) = q^*$. Next suppose $m - 1 \notin T$. Let $\overline{m}$ be the largest signal in T that is smaller than m. Since $q(m') = q^*$ for all $m' \in T$ and $m' \geq m$, we have $W(t_{\overline{m}+1};\overline{m}+1) = W(m';\overline{m}+1)$ for any $t_{\overline{m}+1} \in T_{\overline{m}+1}$. For all $n > \overline{m}+1$, since $W(\overline{m}+1;n) \geq W(t_n;n)$ for $t_n \in T_n$ implies $W(\overline{m}+1;\overline{m}+1) > W(t_n;\overline{m}+1) = W(t_{\overline{m}+1},\overline{m}+1)$, it follows from the refinement that $\hat{q}(\overline{m}+1) = 0$. Given this, if $q(\overline{m}) > q^*$, then $W(\overline{m}+1;n) \geq W(t_n;n)$ for any $n \leq \overline{m}$ and any $t_n \in T_n$ implies $q(\overline{m}+1) > q^*$. It then follows that $W(\overline{m}+1;\overline{m}+1) > W(t_{\overline{m}+1};\overline{m}+1)$, and therefore $q(\overline{m}+1) = 1$ by the refinement, a contradiction. Thus, $q(\overline{m}) = q^*$. □

Proof of Lemma 3. (i), (ii) Suppose $p(m';m) > 0$ for some $m, m' \in T$ and $m' > m \geq l$. By optimality we have $W(m';m) - W(m;m) \geq 0$. Since $q(m) > q(m')$ by Lemma 2, part (ii) of Lemma 1 implies $W(m';n) - W(m;n) > 0$ and hence $p(m;n) = 0$ for all $n < m$. Part (i) of the lemma follows by setting $m = l$ and noting that $p(l;l) = 0$ implies $l \notin T$, a contradiction. Part (ii) follows by noting that for any $m \in T$ and $m > l$, $p(m;m) < 1$ implies that $q(m) = 1$, contradicting Lemma 2.

(iii) By optimality $W(\min T_m; m) \geq W(n;m)$ for all $n \geq \min T_m$. Since from Lemma 2 we have $q(\min T_m) > q(n)$, part (ii) of Lemma 1 implies that $W(\min T_m; m') > W(n;m')$ for all m' such that $m < m' \leq l \leq \min T_m$. Hence $p(n;m') = 0$, and $\max T_{m'} \leq \min T_m$.

(iv) Let $x > 0$ be the largest signal such that $x \notin T$. Note that in any inflationary equilibrium $x < N$. We first show by contradiction that $\hat{q}(x) = 0$. This claim follows from the refinement if $W(x;n) \geq W(t_n;n)$ for any $n > x$ and $t_n \in T_n$ implies that $W(x;x) > W(t_x;x)$ for any $t_x \in T_x$. To establish the latter claim, note that for any $n > x+1$, by optimality $W(x;n) \geq W(t_n;n)$ implies that $W(x;n) \geq W(x+1;n)$. Since $\hat{q}(x) > 0$ and $\hat{q}(x+1) = 0$, part (iii) of Lemma 1 implies that $W(x;x+1) > W(x+1;x+1)$. Since $x+1 \in T_{x+1}$ by (i) and (ii) above, we have $W(x;x+1) > W(t_{x+1};x+1)$ for all $t_{x+1} \in T_{x+1}$, which by optimality implies $W(x;x+1) > W(t_x;x+1)$. Since $t_x \geq x+1$, by part (i) of Lemma 1, we have $W(x;x) > W(t_x;x)$.

Next, we claim that $q(x) = 1$. To see this, note that for each $n < x$ and any $t_n \in T_n$, by optimality $W(x;n) \geq W(t_n;n)$ implies $W(x;n) \geq W(x+1;n)$. Since $\hat{q}(x) = \hat{q}(x+1) = 0$, if $W(x;n) \geq W(x+1;n)$ then $q(x) > q(x+1)$. Since $t_x \geq x+1$, from Lemma 2 we have $q(x) > q(t_x)$. It then follows from part (ii) of Lemma 1 that $W(x;x) > W(t_x;x)$. By the refinement, $q(x) = 1$. Thus there is no $m < x$ such that $m \in T$.

(v) First, consider $\hat{q}(0)$. By (ii) and (iii) above, $p(N;0) > 0$; otherwise, $q(N) = 1$, which is a contradiction. For any $n > 0$ and $t_n \in T_n$, if $W(0;n) \geq W(t_n;n)$ then by optimality $W(0;n) \geq W(N;n)$. By part (i) of Lemma 1, we have $W(0;0) > W(N;0)$. It follows from the refinement that $\hat{q}(0) = 0$.

Next, we show that $\hat{q}(m) = 0$ for any $m = \{1,\ldots,l-1\}$. Suppose instead $\hat{q}(m) > 0$. We show that $W(m;n) \geq W(t_n;n)$ for any $n > m$ and $t_n \in T_n$ implies $W(m;m) > W(t_m;m)$ for any $t_m \in T_m$, which leads to a contradiction by the refinement. First, for any $n > l$, by optimality $W(m;n) \geq W(t_n;n)$ implies $W(m;n) \geq W(l;n)$. Since $\hat{q}(m) > 0$ and $\hat{q}(l) = 0$, by part (iii) of Lemma 1, $W(m;l) > W(l;l)$. Second, for any n such that $m < n \leq l$, we have $t_m \geq n$. If $W(m;n) \geq W(t_n;n)$, optimality implies that $W(m;n) \geq W(t_m;n)$. It then follows from part (i) of Lemma 1 that $W(m;m) > W(t_m;m)$. Combining these two cases, we have $\hat{q}(m) = 0$, as desired.

Finally, consider $q(m)$ for any $m = \{1,\ldots,l-1\}$. Suppose that $W(m;n) \geq W(t_n;n)$ for some $n < m$ and $t_n \in T_n$. By optimality $W(m;n) \geq W(t_m;n)$, which implies $q(m) > q(t_m)$ as $\hat{q}(m) = 0$. It then follows from part (ii) of Lemma 1 that $W(m;m) > W(t_m;m)$. By the refinement, $q(m) = 1$. □

Proof of Proposition 1. Denote by p_n a reporting strategy in state n (i.e., the vector $(p(0;n),\ldots,p(N;n))$), and let P^n be a sequence $(p_n,\ldots,p_N)$. For a given p_n, we use $\overline{t}_n$ ($\underline{t}_n$) to denote the largest (smallest) m such that $p(m;n) > 0$. Although P^n is not a complete strategy, we construct the market beliefs under the assumption that $p(m;n') = 0$ for all m and all $n' < n$ and denote it $q(m \mid P^n)$. When $q(m \mid P^n)$ cannot be obtained using Bayes' rule, it is defined to be 1, while $\hat{q}(m|P^n)$ is defined to be 0. Finally, we denote by $W(m;n \mid P^k)$ the expected payoff to the agency that issues m good ratings and has n clients of good quality when the market beliefs are given by $q(m \mid P^k)$ and $\hat{q}(m \mid P^k)$ for each $m = 0,\ldots,N$.

Definition A.1. We say p_n for some $n>0$ is *compatible with l given P^{n+1}* if it satisfies:

(i) $p(n;n)=1$ in case $n>l$;

(ii) in case $n=l$: (a) $p(m;n)=0$ for all $m<l$; (b) $W(n;n|P^n)=W(t;n|P^n)$ for all $n<t<\overline{t}_n$; (c) $W(n;n|P^n)\leq W(\overline{t}_n;n|P^n)$ if $\overline{t}_n<N$; and (d) $p(N;n)=1-\sum_{m<N}p(m;n)$;

(iii) in case $n<l$: (a) $\overline{t}_{n+1}+1\geq \underline{t}_n\geq \overline{t}_{n+1}$; (b) $W(\underline{t}_n;n|P^n)=W(t;n|P^n)$ for all $\underline{t}_n<t<\overline{t}_n$; (c) $W(\underline{t}_n;n|P^n)\leq W(\overline{t}_n;n|P^n)$ if $\overline{t}_n<N$; (d) $p(N;n)=1-\sum_{m<N}p(m;n)$; (e) $W(\underline{t}_{n'}-1;n'|P^n)\leq W(\underline{t}_{n'};n'|P^n)$ for all $n\leq n'\leq l$ if $\underline{t}_n<N$; (f) $W(\overline{t}_{n+1};n+1|P^n)\geq W(\underline{t}_n;n+1|P^n)$ if $\overline{t}_n>\underline{t}_n$; and (g) $W(\underline{t}_{n'};n'|P^n)=W(\overline{t}_{n'};n'|P^n)$ for all $n'>n$ if $\overline{t}_n>\overline{t}_{n+1}$.

We say P^n is *compatible with l* if p_k is compatible with l given $P^{k+1}=(p_{k+1},\dots,p_N)$ for each k such that $N>k\geq n$, and P^n is *compatible* if it is compatible with some $l=0,\dots,N$.

Lemma A.1. *If $P=(p_0,\dots,p_N)$ is the reporting strategy in an equilibrium with threshold l, then $P^1=(p_1,\dots,p_N)$ is compatible with l.*

Proof. We show that, if P^{n+1} is compatible with l and P is an equilibrium strategy, then p_n is compatible with l given P^{n+1}. The lemma then follows from observing that there is only one way of constructing an equilibrium strategy p_N and it is compatible with all l.

Part (i) of Definition A.1 follows immediately from the definition of equilibrium.

Part (ii). (a) follows immediately from the definition of equilibrium. Next, note that part (iii) of Lemma 3 implies that in any equilibrium with threshold l and each $n\leq l$, $W(t;n|P^n)=W(t;n)$ for all $t<\overline{t}_n$ and $W(\overline{t}_n;n|P^n)\geq W(\overline{t}_n;n)$. Thus by Lemma 3, (b) and (c) must hold in equilibrium. (d) is clearly necessary.

Part (iii). (a) follows from part (iii) and (iv) of Lemma 3. The proof of (b), (c), and (d) is the same as for part (ii) above. Condition (e) is necessary for equilibrium because $W(\underline{t}_{n'}-1;n'|P^n)=W(\underline{t}_{n'}-1;n')$ and $W(\underline{t}_{n'};n'|P^n)\geq W(\underline{t}_{n'};n')$ for all $n\leq n'\leq l$. (f) and (g) follow because $W(m;n'|P^n)=W(m;n')$ for all n' and all $m<\overline{t}_n$. □

Lemma A.1 establishes that compatibility of P^1 is a necessary condition for $P=(p_0,P^1)$ to be an equilibrium. However, it is not sufficient since no restriction is imposed on p_0. Next we give a definition of compatibility of a vector $p_0=(p(0;0),\dots,p(N;0))$ given a compatible P^1. This definition ensures that all the equilibrium conditions are satisfied but does not require $\sum_m p(m;0)=1$ or $p(m;0)\geq 0$ for all $m=0,\dots,N$, so that p_0 may not be a valid reporting strategy.

Definition A.2. Given P^1 compatible with l, we say that p_0 is compatible with P^1 if, when $l>0$:

(i) $p(m;0)=0$ for all $m<\overline{t}_1$;

(ii) $p(\overline{t}_1;0)$ is such that $W(\underline{t}_n;n|p_0,P^1) = W(\overline{t}_n;n|p_0,P^1)$ and $W(\underline{t}_n - 1;n|p_0,P^1) \leq W(\underline{t}_n;n|p_0,P^1)$, for all $n \leq l$;

(iii) $p(m;0)$ for $\overline{t}_1 < m \leq N$ satisfies: (a) if $p(\overline{t}_1;0) > 0$, then $p(m;0)$ is the value such that $W(\overline{t}_1;0|p_0,P^1) = W(m;0|p_0,P^1)$; and (b) if $p(\overline{t}_1;0) = 0$, then $p(\overline{t}_1+1;0)$ satisfies $W(\overline{t}_1;0|p_0,P^1) \leq W(\overline{t}_1+1;0|p_0,P^1)$ and $W(\overline{t}_1;1|p_0,P^1) \geq W(\overline{t}_1+1;1|p_0,P^1)$, and $p(m;0)$ is such that $W(\overline{t}_1+1;0|p_0,P^1) = W(m;0|p_0,P^1)$ for $m > \overline{t}_1+1$;

and, when $l = 0$, $p(0;0) \in [0,1]$ and for each $m > 0$, $p(m;0)$ satisfies (iii) with $\overline{t}_1 = 0$.

By definition, if P^1 is compatible with l and p_0 is compatible with P^1 we have

$$W(\underline{t}_n;n|p_0,P^1) = W(t;n|p_0,P^1) \geq W(m;n|p_0,P^1), \forall n \leq l, m \geq l, \underline{t}_n \leq t \leq \overline{t}_n. \tag{A.1}$$

Moreover, for any vector p_0 that is not compatible with P^1, (A.1) is violated and hence $P = (p_0,P^1)$ is not an equilibrium. By Definitions A.1 and A.2, only $p(N;0)$ can be negative in a compatible vector p_0 and, in that case $p(m;0) = 0$ for each $m < N$. Thus, if $\sum_m p(m;0) = 1$, the vector p_0 is a reporting strategy and $P = (p_0,P^1)$ satisfies all the properties of Lemma 3. It follows that $P = (p_0,P^1)$ is an equilibrium only if p_0 is compatible with P^1 and $\sum_m p(m;0) = 1$. The following lemma verifies that these two conditions are also sufficient for an equilibrium.

Lemma A.2. *Let $P^1 = (p_1,\ldots,p_N)$ be compatible. Then $P = (p_0,P^1)$ is an equilibrium if and only if p_0 is compatible with P^1 and $\sum_m p(m;0) = 1$.*

Proof. Since (A.1) holds, we need to argue only that (a) given P there are no profitable deviations to out-of-equilibrium messages and (b) in all states $n > l$, the agency has no incentive to deviate from the reporting strategy $p(n;n) = 1$.

(a) The most profitable deviation among out-of-equilibrium signals is always $m = l-1$. Part (iii) (e) of Definition A.1 and part (ii) of Definition A.2 imply that

$$W(l;l-1) \geq W(l-1;l-1), \tag{A.2}$$

because either $\underline{t}_{l-1} = l$ or $q(l) = 1$. By part (i) of Lemma 1, (A.2) implies that $W(l;l) > W(l-1;l)$, and, since $q(l-1) \geq q(l)$, by (ii) of Lemma 1 it implies that $W(l;n) \geq W(l-1;n)$ for all $n < l-1$. Finally, since $W(l;l) > W(l-1;l)$ and since $W(l;n) - W(l-1;n) = W(l;l) - W(l-1;l)$ for all $n > l$ because $\hat{q}(l-1) = \hat{q}(l) = 0$, we have $W(l;n) > W(l-1;n)$ for all $n > l$.

(b) The claim is trivial if $l = 0$ since in that case p_0 compatible with P^1 implies that $q(m) = q^*$ for all $m = 1,\ldots,N$. If P^1 is not compatible with $l = 0$, then $q(l) > q^*$ because either $l > 1$ and (A.2) holds, or $l = 1$ and $p(l;l) < 1$, which implies $q(l) = 1$. It follows that $q(m) > q(n) > q^*$ for all $n > m \geq l$. First we show that $W(n;0) - W(m;0) \geq 0$ implies $W(n;n) - W(m;n) \geq 0$. This is trivially true for any m if $n = m$. Now, the condition $W(n;0) - W(m;0) \geq 0$ is equivalent to

$$\frac{U(B,g,q(m)) - U(B,g,q(n))}{U(B,g,q(n)) - U(B,b,0)} \leq \frac{n-m}{m}. \tag{A.3}$$

Similarly, the condition $W(n;n)-W(m;n)\geq 0$ is equivalent to

$$\frac{U(G,g,q(m))-U(G,g,q(n))}{U(G,g,q(n))-U(G,b,0)}\leq\frac{n-m}{m}. \tag{A.4}$$

When Assumption 4 holds, $[U(G,g,q)-U(G,b,0)]/[U(B,g,q)-U(B,b,0)]$ is decreasing in $q\in(q^*,1)$. So,

$$\frac{U(G,g,q(m))-U(G,b,0)}{U(B,g,q(m))-U(B,b,0)}\leq\frac{U(G,g,q(n))-U(G,b,0)}{U(B,g,q(n))-U(B,b,0)}.$$

This condition implies

$$\frac{U(B,g,q(m))-U(B,g,q(n))}{U(B,g,q(n))-U(B,b,0)}\leq\frac{U(G,g,q(m))-U(G,g,q(n))}{U(G,g,q(n))-U(G,b,0)}.$$

Hence, (A.3) implies (A.4). It follows that if $p(m;n)>0$ for some $n>m\geq l$, then $W(m;0)>W(n;0)$. By part (ii) of Lemma 1, $W(m;k)>W(n;k)$ for any $k\leq m$, which contradicts (A.1). □

Let $\mathscr{P}^1$ denote the collection of all sequences of reporting strategies $P^1=(p_1,\dots,p_N)$ that are compatible. The next two lemmas establish that $\mathscr{P}^1$ is non-empty, closed, and connected.

Lemma A.3. *Let $P^{n+1}=(p_{n+1},\dots,p_N)$ be compatible with l. Then the set of reporting strategies in state n compatible with l given P^{n+1} is non-empty, closed, and connected.*

Proof. (i) For $n>l$ the claim is trivially true since by Definition A.1 there is a unique p_n compatible with l given P^{n+1} and it has $p(n;n)=1$.

(ii) For $n=l$, the definition of compatibility implies that p_l is compatible if and only if (a) $p_n(m)=0$ for all $m<l$, (b) $p(l;l)\in[0,1]$, (c) for each $l<m<N$, $p(m;l)$ is equal to the minimum of $1-\sum_{k<m}p(k;l)$ and the value such that $W(l;l|p_l,P^{l+1})=W(m;l|p_l,P^{l+1})$, and (d) $p(N;l)=1-\sum_{k<N}p(k;l)$. Note that $W(l;l|p_l,P^{l+1})$ is independent of p_l. Further, for all $m>l$ the function $W(m;l|p_l,P^{l+1})$ depends only on $p(m;l)$, is strictly decreasing in $p(m;l)$, and satisfies $W(l;l|p_l,P^{l+1})<W(m;l|p_l,P^{l+1})$ when $p(m;l)=0$. Thus, the set of compatible p_l is non-empty and it is closed because each compatible p_l is uniquely determined by the value of $p(l;l)$ and there exists a compatible p_l for each value of $p(l;l)\in[0,1]$. Finally, if p_l is compatible and $\{p_l^i\}_{i=1,2,\dots}$ is a sequence of compatible strategies with $\lim_{i\to\infty}p^i(l;l)=p(l;l)$ then $\lim_{i\to\infty}p^i(m;l)=p(m;l)$ for each $m=0,\dots,N$, hence the set of compatible strategies is connected.

(iii) For $n<l$, the definition of compatibility implies that there is exactly one p_n compatible with l given P^{n+1} if $\overline{t}_{n+1}=N$ or $W(\underline{t}_{n'};n'|P^{n+1})<W(\overline{t}_{n'};n'|P^{n+1})$ for some $n<n'$. Otherwise, p_n is compatible if and only if

(a) for each $m<\overline{t}_{n+1}$, $p(m;n)=0$

(b) $p(\overline{t}_{n+1};n)$ is non-negative and not larger that the minimum of 1 and the largest value such that $W(\underline{t}_{n'}-1;n'\mid P^n)\leq W(\underline{t}_{n'};n'\mid P^n)=W(\overline{t}_{n'};n'\mid P^n)$ for all $n<n'\leq l$

(c) if $p(\bar{t}_{n+1};n)>0$ then

(c-i) $p(m;n)$ is the minimum of $1-\sum_{k<m}p(k;n)$ and the value such that $W(\bar{t}_{n+1};n\mid P^n)=W(m;n\mid P^n)$ for each $\bar{t}_{n+1}<m<N$ and

(c-ii) $p(N;m)=1-\sum_{k<N}p(k;n)$

(d) if $p(\bar{t}_{n+1};n)=0$ then

(d-i) $p(\bar{t}_{n+1}+1;n)$ is at least as large as the minimum of 1 and the value such that $W(\bar{t}_{n+1};n+1\mid P^n)=W(\bar{t}_{n+1}+1;n+1\mid P^n)$ and it is at most as large as the minimum of 1 and the value such that $W(\bar{t}_{n+1};n\mid P^n)=W(\bar{t}_{n+1}+1;n\mid P^n)$

(d-ii) for each $\bar{t}_{n+1}+1<m<N$, $p(m;n)$ is the minimum of $1-\sum_{k<m}p(m;n)$ and the value such that $W(\bar{t}_{n+1}+1;n\mid P^n)=W(m;n\mid P^n)$

(d-iii) $p(N;n)=1-\sum_{k<N}p(k;n)$.

By the compatibility of P^{n+1}, $p(\bar{t}_{n+1};n)=0$ satisfies (b). Hence the set of all values of $p(\bar{t}_{n+1};n)$ that satisfy (b) is non-empty, closed, and connected. Further, for each $p(\bar{t}_{n+1};n)>0$ that satisfies (b), $p(m;n)$ is uniquely determined by (c) for each $m>\bar{t}_{n+1}$. When $p(\bar{t}_{n+1};n)=0$ instead, if $\bar{t}_{n+1}+1=N$, $p(\bar{t}_{n+1}+1;n)$ is uniquely determined equal to 1 by (d-iii). Otherwise, note that $q(m\mid P^n)$ depends only on $p(m;n)$, is strictly decreasing in $p(m;n)$, and satisfies $q(m\mid P^n)=1$ when $p_n(m)=0$ for each $m>\bar{t}_{n+1}$. Further, by Lemma 1, the value such that $W(\bar{t}_{n+1};n+1\mid P^n)=W(\bar{t}_{n+1}+1;n+1\mid P^n)$ is smaller than the value such that $W(\bar{t}_{n+1};n\mid P^n)=W(\bar{t}_{n+1}+1;n\mid P^n)$. It follows that the set of values of $p(\bar{t}_{n+1}+1;n)$ that satisfy (d-i) is non-empty, closed, and connected, and for each such value, $p(m;n)$ is uniquely determined by (d-ii) and (d-iii) for all $m>\bar{t}_{n+1}+1$. Finally, when $p(\bar{t}_{n+1}+1;n)$ assumes the maximum value that respects condition (d-i), the resulting p_n also satisfies condition (c). □

Lemma A.4. *The set $\mathscr{P}^1$ is non-empty, closed, and connected.*

Proof. An immediate implication of Lemma A.3 and the definition of compatibility is that, for each l, the collection of all P^1 compatible with l is non-empty, closed, connected, and completely ordered by $>_{LF}$. We show that for each $0<l<N$, the smallest P^1 compatible with l is also compatible with $l-1$. The claim then follows from observing that the largest P^1 compatible with $N-1$ has $p(N;n)=1$ for all $n=1,\ldots,N$, which is the only sequence of reporting strategies compatible with N.

Let P^1 be a reporting strategy compatible with l such that $p(l;l)=1$ and $p(l;l-1)$ is the minimum of 1 and the value such that $W(l;l-1\mid P^{l-1})=W(l-1;l-1\mid P^{l-1})$. This implies that P^{l-1} also is compatible with $l-1$. The lemma follows by noting that for all $n<l-1$ the definition that p_n is compatible with l is identical to the definition that p_n is compatible with $l-1$. □

Next, consider the correspondence $\phi:\mathscr{P}^1\to\mathbb{R}$ defined by

$$\phi(P^1)=\left\{x\in\mathbb{R}\mid x=\sum_m p(m;0)\text{ for some }p_0\text{ compatible with }P^1\right\}.$$

For each compatible P^1, the set $\phi(P^1)$ is obtained by first finding all vectors p_0 compatible with P^1 and then by summing over all entries for each such vector. By Lemma A.2, for any equilibrium $P=(p_0,P^1)$, we have $\phi(P^1)\ni 1$. Further, if $\phi(P^1)\ni 1$ for some $P^1\in\mathscr{P}^1$, then there exists p_0 such that $P=(p_0,P^1)$ is an equilibrium. The following properties of ϕ are used to conclude the proof of the proposition.

Lemma A.5. *For all $P^1\in\mathscr{P}^1$, the set $\phi(P^1)$ is closed and convex, and ϕ is upper hemicontinuous.*

Proof. First we establish that $\phi(P^1)$ is closed and convex. Let P^1 be compatible with l. We distinguish three cases.

(i) If $l=0$, then $p(n;n)=1$ for all $n=1,\dots,N$ and by Definition A.2, p_0 is compatible if and only if $p(0;0)\in[0,1]$ and $p(m;0)=(1-q^*)\pi_m/(q^*\pi_0)$ for all $m=1,\dots,N$.

(ii) If $l=N$, then $p(N;n)=1$ for all $n=1,\dots,N$ and by Definition A.2, p_0 is compatible if and only if $p(m;0)=0$ for all $m=0,\dots,N-1$, and $p(N;0)$ is not larger that the value such that $W(N-1;N-1\mid p_0,P^1)=W(N;N-1\mid p_0,P^1)$. Such a value exists and is unique since $W(N;N-1\mid p_0,P^1)$ is continuous and strictly decreasing in $p(0;0)$, $W(N;N-1\mid p_0,P^1)>W(N-1;N-1\mid p_0,P^1)$ in the limit as $p(N;0)\to-\infty$, and $W(N;N-1\mid p_0,P^1)<W(N-1;N-1\mid p_0,P^1)$ in the limit as $p(N;0)\to+\infty$.

(iii) If $0<l<N$, an argument analogous to that in part (iii) of the proof of Lemma A.3 can be used to establish that the set of vectors p_0 compatible with P^1 is nonempty, closed, connected, and compact. Thus, the function $\sum_m p(m;0)$ on the set of compatible vectors p_0 has a minimum and maximum and assumes all values in between.

To prove that ϕ is upper hemicontinuous by contradiction, we assume that there is a sequence of $P^1(i)$ with $\lim_{i\to\infty}P^1(i)=P^1$, and a sequence of $p_0(i)$ with $\lim_{i\to\infty}p_0(i)=p_0$, such that $p_0(i)$ is compatible with $P^1(i)$ for all i but p_0 is not compatible with P^1. By definition of compatibility, there exists n such that $W(t_n;n\mid p_0,P^1)<W(m;n\mid p_0,P^1)$ with $t_n\in T_n$. This implies that for i large enough, $W(t_n;n\mid p_0(i),P^1(i))<W(m;n\mid p_0(i),P^1(i))$ and $t_n\in T_n(i)$, a contradiction of compatibility. □

To complete the proof of the proposition, note that the reporting strategy P^1 defined by $p(n;n)=1$ for all $n=1,\dots,N$ is compatible. By Definition A.2, there is a vector p_0 compatible with P^1 such that $p(0;0)=1$ and $p(m;0)>0$ for all $m>1$. Thus, $\max\phi(P^1)>1$. On the other hand, the reporting strategy P^1 defined by $p(N;n)=1$ for all $n=1,\dots,N$ is also compatible. By Definition A.2, any vector p_0 with $p(m;0)=0$ for all $m<N$ and $p(N;0)$ sufficiently small is compatible with P^1, and therefore $\phi(P^1)$ is unbounded from below. Since ϕ is upper hemicontinuous, there exists some P^1 such that $\phi(P^1)\ni 1$, establishing the existence of an inflationary equilibrium. To establish a sufficient condition for an equilibrium to have threshold $l>0$, note that the only P^1 compatible with $l=0$ has $p(n;n)=1$ for all $n=1,\dots,N$, and the minimum of $\phi(P^1)$ is achieved by $p(0;0)=0$ and $p(m;0)=(1-q^*)\pi_m/(q^*\pi_0)$. Thus, if $\pi_0<1-q^*$, there is an inflationary equilibrium with $l>0$. □

Proof of Lemma 4. First, we show that if X is exchangeable and MTP_2, then, for any i, $(X_i,Y_i,Z_i,\tilde{Z}_i)$ is MTP_2. If X is exchangeable and MRR_2, then, for any i, $(X_i,Y_i,N-1-Z_i,N-1-\tilde{Z}_i)$ is MTP_2.

Since X is exchangeable, for any two realizations x and x' of X such that $\sum_i x_i = \sum_i x'_i$, we have $f(x)=f(x')$. Let f_n represent the probability of x such that $\sum_i x_i=n$.

Assume that f is MTP_2. Let $h(x_i,z_i)$ be the joint probability function of (X_i,Z_i). For $z_i\geq z'_i$, we have

$$\begin{aligned}h(1,z_i)h(0,z'_i)&=\binom{N-1}{z_i}\binom{N-1}{z'_i}f_{z_i+1}f_{z'_i}\\&\geq\binom{N-1}{z_i}\binom{N-1}{z'_i}f_{z_i}f_{z'_i+1}=h(1,z'_i)h(0,z_i).\end{aligned}$$

Thus, $h(x_i,z_i)$ is also MTP_2.

Now consider the conditional distributions $\phi(y_i\mid x_i)$ and $\psi(\tilde{z}_i\mid z_i)$. These are

$$\phi(y_i\mid x_i)=\begin{cases}0 & \text{if } y_i<x_i\\ 1 & \text{if } y_i=x_i=1\\ \beta(1-x_i,y_i-x_i,p) & \text{otherwise}\end{cases}$$

$$\psi(\tilde{z}_i\mid z_i)=\begin{cases}0 & \text{if } \tilde{z}_i<z_i\\ 1 & \text{if } \tilde{z}_i=z_i=N-1\\ \beta(N-1-z_i,\tilde{z}_i-z_i,p) & \text{otherwise.}\end{cases}$$

It is straightforward to verify that both ϕ and ψ are MTP_2. The joint distribution of $(X_i,Y_i,Z_i,\tilde{Z}_i)$ is simply $h(x_i,z_i)\phi(y_i\mid x_i)\psi(\tilde{z}_i\mid z_i)$. Since each of these component functions is MTP_2, the joint distribution is MTP_2. When f is MRR_2, using a reasoning similar to the one above we can establish that the joint distribution of $(X_i,Y_i,N-1-Z_i,N-1-\tilde{Z}_i)$ is also MTP_2.

By the above result, if f is MTP_2, then $(X_i,Y_i,Z_i,\tilde{Z}_i)$ is MTP_2. Since the marginal distribution of any subset of a MTP_2 vector is itself MTP_2 (Karlin and Rinott 1980, Proposition 3.1), this means that the joint distribution of $(X_i,Y_i,\tilde{Z}_i)$ is MTP_2. Suppose $h^*(\tilde{z}_i\mid x_i,y_i)$ represents the conditional probability function of $\tilde{Z}_i$ given X_i and Y_i. The MTP_2 property of the joint distribution implies that, for any $\tilde{z}_i\geq\tilde{z}'_i$, we have $h^*(\tilde{z}_i\mid 1,1)h^*(\tilde{z}'_i\mid 0,1)\geq h^*(\tilde{z}_i\mid 0,1)h^*(\tilde{z}'_i\mid 1,1)$, and thus the likelihood ratio $h^*(\cdot\mid 1,1)/h^*(\cdot\mid 0,1)$ is monotone increasing, implying that the distribution $\{r^G(m)\}$ first-order stochastically dominates $\{r^B(m)\}$.

When f is MRR_2, the joint distribution of $(X_i,Y_i,N-1-\tilde{Z}_i)$ is MTP_2. This implies that the likelihood ratio $h^*(\cdot\mid 1,1)/h^*(\cdot\mid 0,1)$ is monotone decreasing. Hence, the distribution $\{r^B(m)\}$ first-order stochastically dominates $\{r^G(m)\}$. □

PROOF OF LEMMA 5. Suppose f is MTP_2. Let $h^{**}(x_i \mid y_i, \tilde{z}_i)$ represent the conditional probability function of X_i given Y_i and $\tilde{Z}_i$. By the same argument as in Lemma 4, the joint distribution of these three variables is MTP_2. Therefore, for $m \geq m'$, the conditional distribution satisfies

$$h^{**}(1 \mid 1, m-1) h^{**}(0 \mid 1, m'-1) \geq h^{**}(0 \mid 1, m-1) h^{**}(1 \mid 1, m'-1).$$

This condition implies that $q(m) = h^{**}(1 \mid 1, m-1) \geq h^{**}(1 \mid 1, m'-1) = q(m')$.

When f is MRR_2, we have $q(m') \geq q(m)$ for $m \geq m'$, as

$$h^{**}(1 \mid 1, m'-1) h^{**}(0 \mid 1, m-1) \geq h^{**}(0 \mid 1, m'-1) h^{**}(1 \mid 1, m-1). \qquad \square$$

PROOF OF LEMMA 6. For each $m = 1, \ldots, N$,

$$\begin{aligned} r^B(m) &= \sum_{n=0}^{N-1} \Pr[\tilde{Z}_i = m-1 \mid X_i = 0, Y_i = 1, Z_i = n] \Pr[Z_i = n \mid X_i = 0, Y_i = 1] \\ &= \sum_{n=0}^{m-1} \beta(N-n-1, m-n-1, p) \frac{\pi_n}{1-\pi} \frac{N-n}{N} \\ &= \sum_{n=0}^{m} \pi_n \beta(N-n, m-n, p) \frac{m-n}{N(1-\pi)p}. \end{aligned}$$

For each $m = 1, \ldots, N$, define

$$\begin{aligned} \mathcal{N}(m) &= \sum_{n=0}^{m} \pi_n \beta(N-n, m-n, p) n \\ \mathcal{D}(m) &= \sum_{n=0}^{m} \pi_n \beta(N-n, m-n, p) m. \end{aligned}$$

Since $\sum_{m=1}^{N} r^B(m) = 1$, we have

$$\sum_{m=1}^{N} \Big(\mathcal{D}(m) - \mathcal{N}(m) \Big) = N(1-\pi)p.$$

Note that since $r^G(m) = \mathcal{N}(m)/(\pi N)$, from $\sum_{m=1}^{N} r^G(m) = 1$ we have

$$\sum_{m=1}^{N} \mathcal{N}(m) = N\pi,$$

and hence

$$\sum_{m=1}^{N} \mathcal{D}(m) = N(\pi + (1-\pi)p).$$

Finally, note that from equation (6) we have $q(m) = \mathscr{N}(m)/\mathscr{D}(m)$. Since $r^B(m) = (\mathscr{N}(m) - \mathscr{D}(m))/(N(1-\pi)p)$, we have

$$\sum_{m=1}^{N} r^B(m)q(m) = \frac{1}{N(1-\pi)p}\sum_{m=1}^{N}\mathscr{D}(m)q(m)(1-q(m)).$$

Since $q(1-q)$ is concave in q, Jensen's inequality implies that the above expression is less than or equal to

$$\frac{\sum_{m=1}^{N}\mathscr{D}(m)}{N(1-\pi)p}\left(\frac{\sum_{m=1}^{N}q(m)\mathscr{D}(m)}{\sum_{m=1}^{N}\mathscr{D}(m)}\right)\left(1-\frac{\sum_{m=1}^{N}q(m)\mathscr{D}(m)}{\sum_{m=1}^{N}\mathscr{D}(m)}\right).$$

The lemma follows immediately. □

Lemma A.6. *For any small and positive* ϵ,

$$\lim_{N\to\infty}\sum_{n\in[N(\pi-\epsilon),N(\pi+\epsilon)]}\;\sum_{m>N(\pi+\epsilon)}\pi_n^N p^N(m;n) = 0,$$

where $\pi_n^N = (1-\alpha)\beta(N,n,\pi)$.

Proof. The claim is trivially true if $\alpha = 1$. Assume $\alpha < 1$. Suppose that for some small $\epsilon > 0$ there exists a subsequence in N such that

$$\lim_{N\to\infty}\sum_{n\in[N(\pi-\epsilon),N(\pi+\epsilon)]}\;\sum_{m>N(\pi+\epsilon)}\pi_n^N p^N(m;n) > 0.$$

Since $\sum_{n\notin[N(\pi-\gamma),N(\pi+\gamma)]}\pi_n^N$ equals $\alpha < 1$ in the limit for any $\gamma > 0$,

$$\lim_{N\to\infty}\sum_{n\in[N(\pi-\gamma),N(\pi+\gamma)]}\;\sum_{m>N(\pi+\epsilon)}\pi_n^N p^N(m;n) > 0.$$

Define the collection of messages

$$\mathscr{M}^N = \left\{m > (\pi+\epsilon)N \mid p^N(m;n) > 0 \text{ for some } n \in [N(\pi-\gamma), N(\pi+\gamma)]\right\}.$$

The set $\mathscr{M}^{\mathscr{N}}$ contains all messages larger that $N(\pi+\epsilon)$ that are sent with positive probability in some state n close to πN. We claim that $N \notin \mathscr{M}^{\mathscr{N}}$ for sufficiently large N; otherwise, $p^N(N;0) = 1$ by Lemma 3 and we would have a sequence of equilibria with $q^N(N)$ approaching π and $q^N(n)$ for n close to $N\pi$ approaching 1, which contradicts condition (11). Now, since

$$\lim_{N\to\infty}\sum_{n>N(\pi+\gamma)}\;\sum_{m\in\mathscr{M}^{\mathscr{N}}}\pi_n^N p^N(m;n) = 0,$$

we have

$$\lim_{N\to\infty}\sup_{m\in\mathscr{M}^{\mathscr{N}}}\frac{\sum_{n<N(\pi+\gamma)}\pi_n^N p^N(m;n)}{\sum_n \pi_n^N p^N(m;n)} = 1.$$

For any $m \in \mathscr{M}^{\mathscr{N}}$, the equilibrium belief $q(m)^N$ is given by

$$\frac{\sum_{n<N(\pi+\gamma)} \pi_n^N p^N(m;n) n/m + \sum_{n>N(\pi+\gamma)} \pi_n^N p^N(m;n) \min\{n/m, 1\}}{\sum_n \pi_n^N p^N(m;n)}$$
$$\leq \frac{((\pi+\gamma)N/m) \sum_{n<N(\pi+\gamma)} \pi_n^N p^N(m;n) + \sum_{n>N(\pi+\gamma)} \pi_n^N p^N(m;n)}{\sum_n \pi_n^N p^N(m;n)}.$$

Thus

$$\lim_{N\to\infty} \inf_{m\in\mathscr{M}^{\mathscr{N}}} m q^N(m) \leq N(\pi+\gamma).$$

Since $q^N(n)$ becomes arbitrarily close to 1 for all n close to $N\pi$, for γ sufficiently small and N sufficiently large there exists $m \in \mathscr{M}^{\mathscr{N}}$ such that by condition (11), $W(n;n) > W(m;n)$ for all $n \in [N(\pi-\gamma), N(\pi+\gamma)]$, a contradiction. □

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